

Math 118-Quiz #2
February 7, 2011

Name: _____

Dunkin' Starbucks
(All you ever wondered about coffee)

Suppose that when a cup of coffee is served to us its temperature is 90°C (90 degrees Celsius). We are told that it will cool at a rate of

$$-7e^{-t/10} \text{ }^\circ\text{C}/\text{minute}.$$

We want to determine such things as what the temperature will be 10 minutes after serving, what the temperature will be at *any* given time t , what the average temperature is during the first 10 minutes after serving, etc.

The various parts that follow are intended to lead you through a discussion of these things. Denote the temperature of the coffee t minutes after serving by $f(t)$.

- a) What is the derivative $f'(t)$ of the temperature function?
- b) What does the Fundamental Theorem of Calculus say about the difference $f(10) - f(0)$?
- c) Use your calculator, with say 100 points of subdivision, to compute the integral

$$\int_0^{10} e^{-t/10} dt.$$

Now what is $\int_0^{10} -7e^{-t/10} dt$?

d) Use b), c) and the data given about the initial temperature to determine the temperature of the coffee 10 minutes after it was served.

e) Differentiate the function

$$70e^{-t/10}$$

with respect to t and compare the result with your observation in part a). Conclude that $f(t) = 70e^{-t/10} + K$ for some constant K . (Explain why this is so.)

f) Finally use the (known) value $f(0)$ to find that particular constant K that gives $f(t)$. You should now have an explicit formula for $f(t)$,

g) Verify that the formula for $f(t)$ that you have found in f) gives the same value when $t = 10$ as that which you found in d),

h) What was the average temperature for the 10 minutes following the time when the coffee was served? What was the average rate of cooling?

