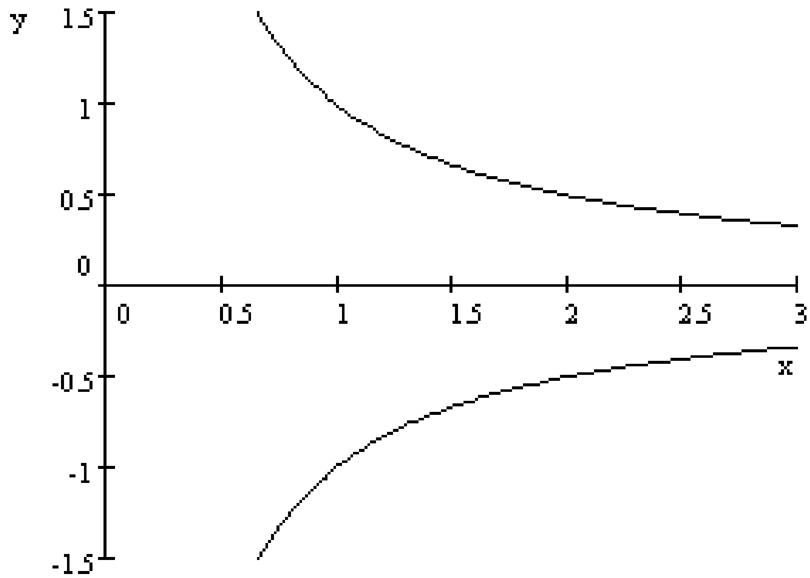
# How to paint an infinite area with a finite amount of paint.

The region to be painted: Consider the region bounded by the two curves $y = 1/x$ and $y = -1/x$, for $x \geq 1$.




**a)** What integral represents the area of this region

**i)** for $1 \leq x \leq 100$? ____________________

**ii)** for $1 \leq x \leq 1000$? ____________________

**iii)** for $1 \leq x \leq b$? ____________________

**b)** Evaluate the integrals in a).

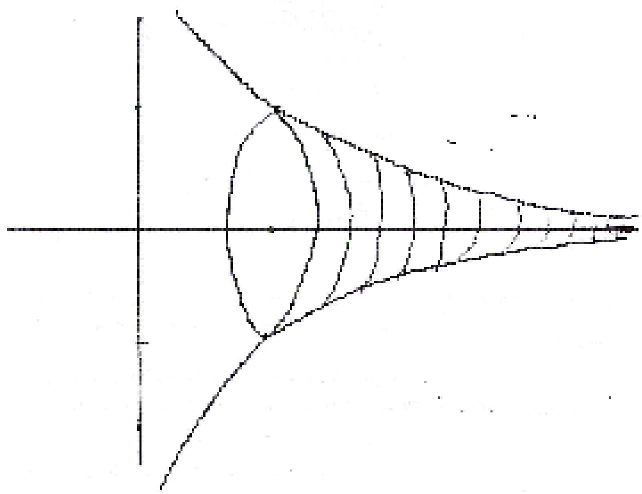
**i)** ____________________

**ii)** ____________________

**iii)** ____________________

**c)** Define the (total) area (for $1 \leq x < \infty$) to be $\int_1^\infty \frac{1}{x} dx$. Is this integral infinite (in value) or finite?

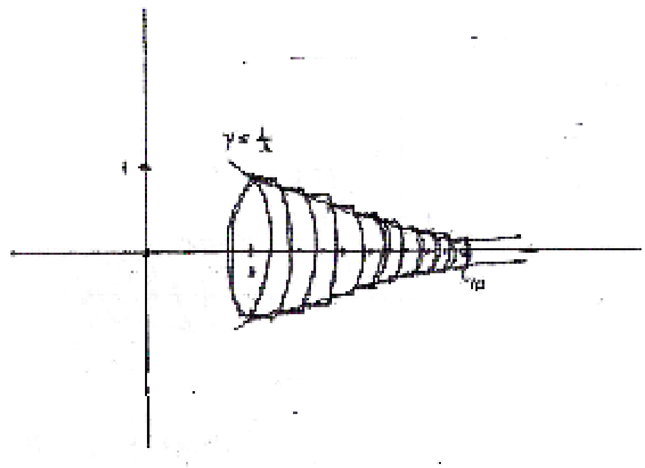
Now, a volume problem (The Paint Bucket): Rotate the curve $y = 1/x$, $x \geq 1$ about the $x$-axis, This sweeps out a surface like that of a vase (for flowers with very long stems).



A vase to be used as paint bucket.

Question: What is the volume of this vase? That is, what is the volume of the region enclosed by this surface?

Here's what we do, here's our approach: We approximate the volume, at least for $x$ ranging from 1 to 10, by thinking of the region as being composed of many thin circular disks stuck together:




For example, disk $i$ has radius $1/x_x$ and thickness $\Delta x$, so has volume

$$\pi \left( \frac{1}{x_i} \right)^2 \Delta x$$

so the total volume is approximated by the sum

$$V \sim \sum_{i=1}^{10} \pi \left( \frac{1}{x_i} \right)^2 \Delta x.$$

**a)** The sum giving the approximate value of the volume looks like a Riemann Sum. What is the corresponding integral?

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**b)** Evaluate this integral:

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**c)** Now consider the case where $1 \le x \le 100$:

**i)** What is the approximating Riemann Sum?

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**ii)** What is the corresponding integral?

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**iii)** Value of the integral: ______________________________

**d)** Answer the same questions as in c) but with $1 \le x \le 1000$.

**i)** Riemann sum : ______________________________

**ii)** Corresponding integral: ______________________________

**iii)** Value of the integral: ______________________________

**e)** If we define the actual volume (as opposed to the approximate volume) to be

$$\begin{aligned} V &= \lim_{b\to\infty} (\text{Approximation for } 1 \le x \le b) \\ &= \lim_{b\to\infty} \left( \pi \int_1^b \left( \frac{1}{x^2} \right) dx \right) \end{aligned}$$

what value do you get?

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**f)** How much red paint (suppose the units are gallons) do we need to fill this vase?

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Finally, to paint our region red, fill your vase with red paint, then carefully dip the region down (vertically) into the center of the vase. (It will just fit nicely.) Remove and let dry. Though the area is infinite, this requires only $\pi$ galllons or so of paint.

SHAZAM!

Success is ours!