Reading Questions for Section 4.5 of Mitzenmacher-Upfal

1. What do you think the book means when it describes a network as sparse? You can google this if you need to. Bonus if you mention the adjacency matrix.
2. The book discusses parallel routing algorithms. Where is the parallelization happening? If the network has n nodes and m edges, and the routing problem is a permutation routing, how many things are happening in parallel at each step? Why?
3. Draw a K5 with nodes A, B, C, D, E. Consider the following packet routing problem, where each a pair x y means the packet starts at x and wants to get to y. Why is this routing problem solvable in 1 parallel step?

A C

B E

C A

D B

E D

1. The book says “Note that the total number of directed edges in the n-cube is 2nN.” Based on Figure 4.1, write out the number of *directed* edges for n = 1, 2, 3, 4 (that is, count each edge you see twice, because it can be traversed either way). Now write out the formula 2nN, where N = 2n. Do they agree? What should the book have put, instead of that coefficient of 2?
2. How can a simple bit-fixing algorithm lead to congestion on the hypercube? What does congestion even mean in this context?
3. Lemma 4.13 is written as “$\leq $”, but in general the two sides will be very far from equal. What would it mean, in terms of packets, edges, and waiting, if the two sides were equal? Note that this would mean no steps were ever taken in parallel, as far as packet M is concerned.
4. Fix a path P. What is the difference between calling a packet “active at a node v” vs. just calling it “active”?
5. Are any properties of the path P used in the analysis of E[H] on page 76? If not, then this analysis holds for *any* path P.
6. For each i, we let (on page 76) j be the first bit needing to be fixed to pass from vi-1 to vi. The book says there are no more than 2j-1 active packets at vi-1 on the path P = (v0, v1,v2, …, vm). Why is this?
7. The book says that for each potentially active packet at vi-1, the probability it’s actually active is 2-(j-1). Why?
8. In the E[H] line, why is m $\leq $ n?
9. Why is P(A) $\leq $ P(B) + P(A | $\overbar{B}$)? Hint: recall that probabilities are between 0 and 1.
10. Where does the book get 30*n* from in the bound for T1(P) on page 76? Hint: it might help to take this on faith for now, read ahead to the analysis, and then come back and try to answer this.
11. Match up the A’s and B’s from the inequality above the line with the 30*n* with the probabilities in the line with the 30*n*.
12. What is a success and what is a failure? Explain the claim that the probability that the active packets cross edges of P more than 30n times is less than the probability that a fair coin flipped 36n times comes up heads fewer than 6n times, where heads corresponds to success.
13. The book claims that “once there have been 6n successes we know there are no more active packets left that can cross an edge of the path.” Explain why this is so. Hint: think of the conditioning event.
14. Explain why knowing that Pr(Z $\leq $ 6n) $\leq 2^{-3n-1}$ implies Pr($T\_{1}\left(P\right)\geq 30n)\leq 2^{-3n}$
15. I claim that the union bound is being used in the proof, towards the bottom of page 77. Identify where, and write out the union in question explicitly.
16. Explain the main differences between the butterfly network and the hypercube network, that break the proof strategy from 4.5.1.
17. Which better describes the routing policy in Theorem 4.14? “The farther the packet has come the (circle one: longer shorter ) it has to wait”
18. Explain why, on page 80, 2n x 3-2n = O(N-1)
19. In Stage II, why is it that in each step a queue receives at most 1 new packet?
20. The proof of Lemma 4.15 finishes by appealing to the first part of the Lemma. How can that first inequality be used to prove the opposite inequality?
21. On page 82, where did the 5*n* term come from? Where did the 40*n* come from?