EAPSI: Studying the Extent to Which Localization Preserves Algebraic Structure in a Categorical Setting

Mathematics at its most basic level is the study of abstract thinking. Category theory follows this approach, and interprets all branches of mathematics as the study of objects and the relationships between them. Localization is a fundamental tool in mathematics which allows one to zoom in on the pertinent information in a problem. In the context of category theory, localization is a way to view two different objects as equivalent. As humans we do this all the time. For example, if two driving routes take the same amount of time we might view them as equivalent. In order to best study this localization procedure, we work in the setting of special kinds of categories called model categories. Such categories admit a special kind of localization called Bousfield localization. Algebra is a powerful computational tool which may be applied in the setting of model categories via objects called operads. The research in this proposal studies the extent to which Bousfield localization preserves algebraic structure. The applications of this work will strengthen the developing bond between homotopy theory and categorical algebra, will allow tools from each area to be applied in the other, and will lead to a better understanding of both fields. The generality of this approach allows for applications in many different fields of mathematics, physics, and computer science.

This research will be conducted jointly with Michael Batanin of Macquarie University. His recent work allows for Bousfield localization to be applied to operad-algebras. The Principal Investigator's recent work allows operad-algebras to be studied after Bousfield localization has been applied. The project will attempt to link up these two approaches, to relate their outputs, to pool the two proof strategies, and to reduce the conditions required so that this work will apply to a wider class of model categories. This NSF EAPSI award is funded in collaboration with the Australian Academy of Science.