Knot Mosaics

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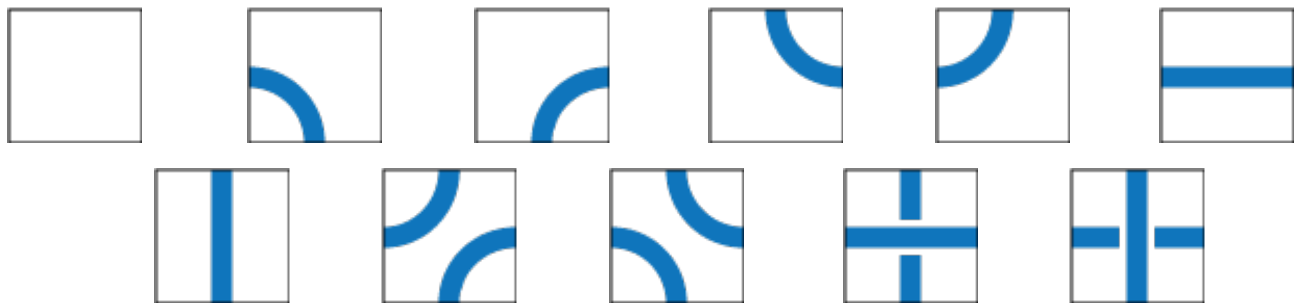
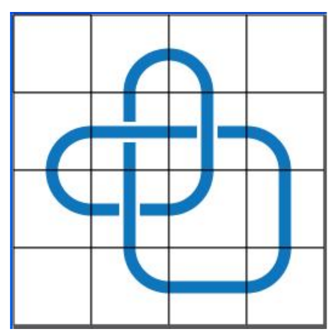
What are Knot Mosaics?

- Lomonaco and Kauffman (2008)





Trefoil

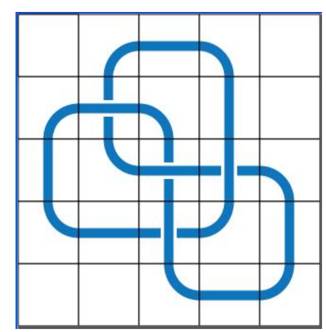


Figure 8

- Kuriya (2008), Shebab (2012) tame knot theory and mosaic knot theory are equivalent

Why I like studying knot mosaics.

- You don’t need to know a lot knot theory
- Wide area of math
  - Knot theory (Tait’s conjecture)
  - Graph theory
  - Combinatorics
  - Algorithms
- Great for undergraduate research

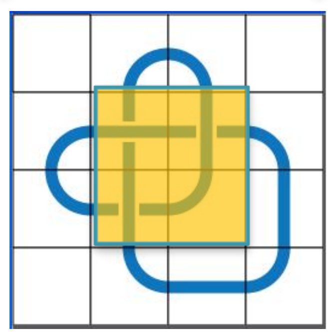
Some terminology

- <u>mosaic number</u> of a knot *K*, denoted *m(K)*

The minimal size mosaic board that a knot will fit on

4

5 → 4?



Trefoil

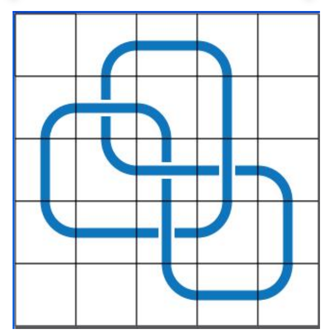
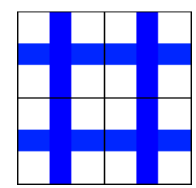


Figure 8

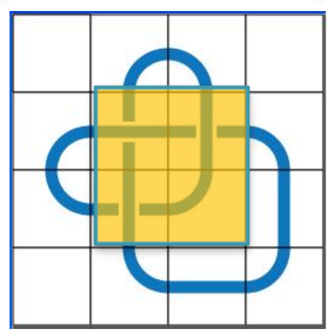


Some terminology

- mosaic number of a knot K, denoted $m(K)$

The minimal size mosaic board that a knot will fit on

$m(3_1)=4$



Trefoil

$m(4_1)=5$

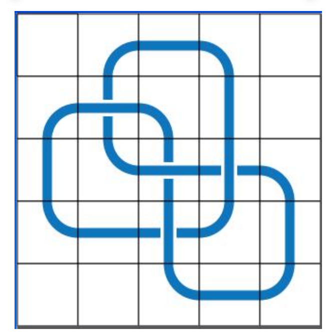
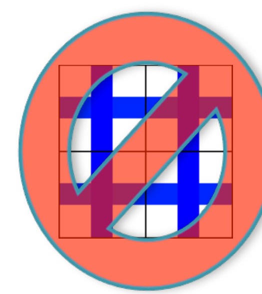
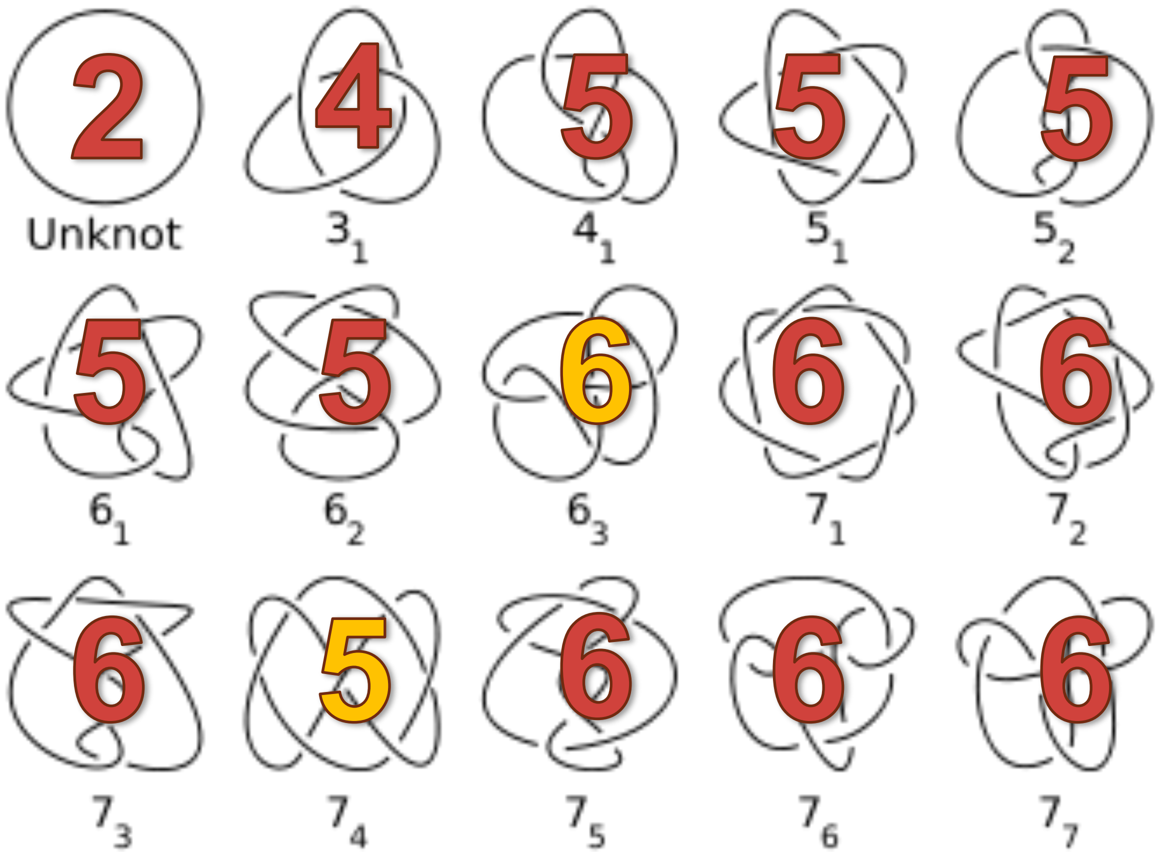


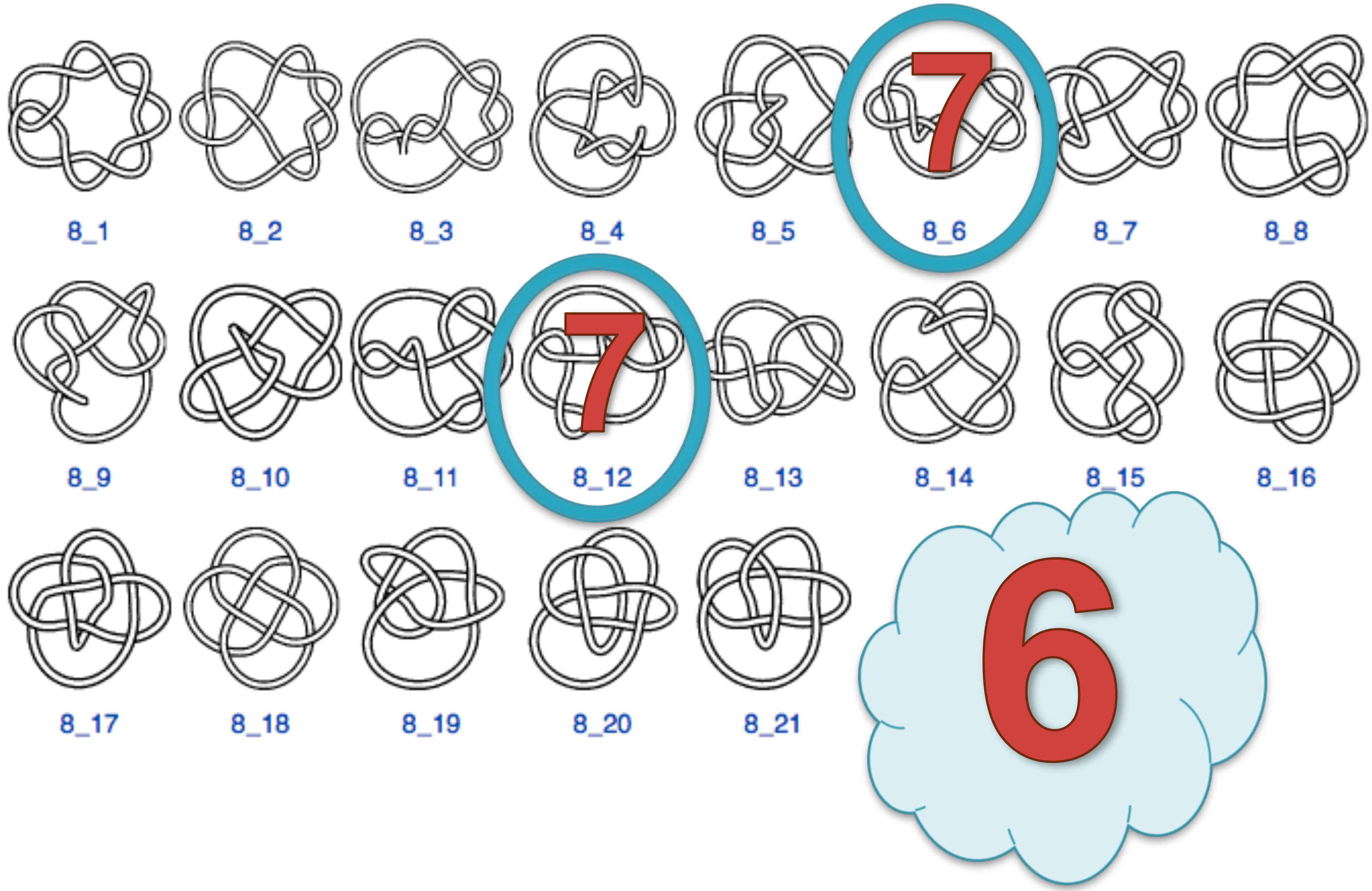
Figure 8



What we know – mosaic number



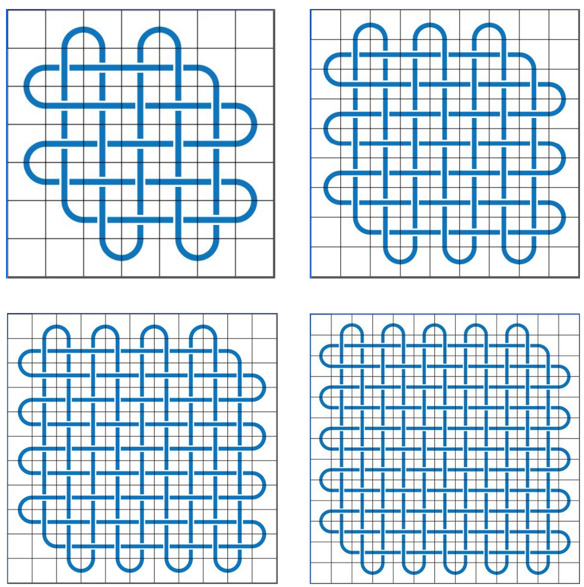
What we ^almost know – mosaic number



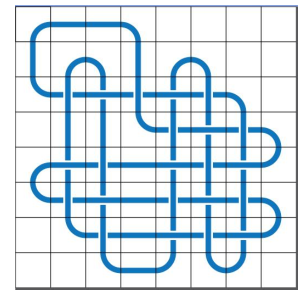
Other things we know about mosaic number…

Do we know the mosaic number for an infinite family of knots?

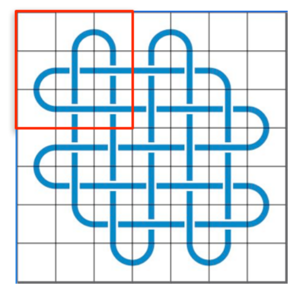
- (L, J. Paat, and E. Evans, 2013)



m(L)=2n+1



Number of Crossings: 22
Mosaic Size: 8



Number of Crossings: 23
Mosaic Size: 7

(Adams) Is there an infinite family of knots whose mosaic number is realized only when the crossing number is not?

Other things we know about mosaic number…

Do we know the mosaic number for an infinite family of knots?

- (L. & Wu, 2012)

$$m(T_{(p,p+1)}) \leq 2p$$

- (H.J. Lee, K. Hong, H. Lee, and S. Oh, 2013)

$$m(T_{(p,q)}) \leq p+q-2 \quad |p-q| \neq 1$$

Other things we know

(Lomonaco and Kauffman)

Is the mosaic number, $m(K)$, related to the crossing number, $c(K)$, of a knot K?

(H.J. Lee, K. Hong, H. Lee, and S. Oh)

- $m(K) \leq c(K)+1$, non-trival knot
- $m(K) \leq c(K)-1$, prime and non-alternating

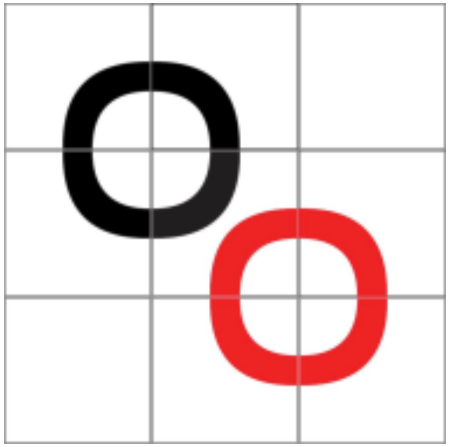
(H. Howards and A. Kobin)

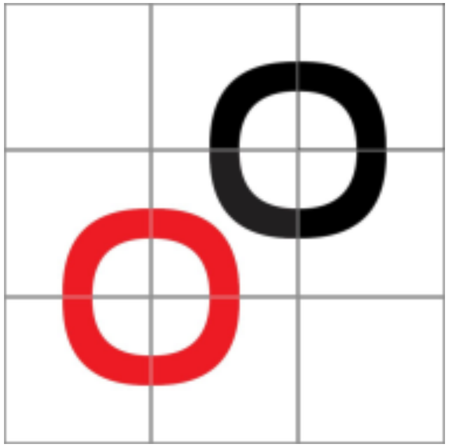
- $c(k) \leq (m(K)-2)^2-2$, m odd
- $c(k) \leq (m(K)-2)^2-(m-3)$, m even

What we're working on – counting conformations

- L., Paat and Shapiro, 2010

Board Size	Conformations
1	1
2	2
3	22
4	2,594
5	4,183,954
6	101,393,411,126





H.J. Lee et al., 2013, 2014

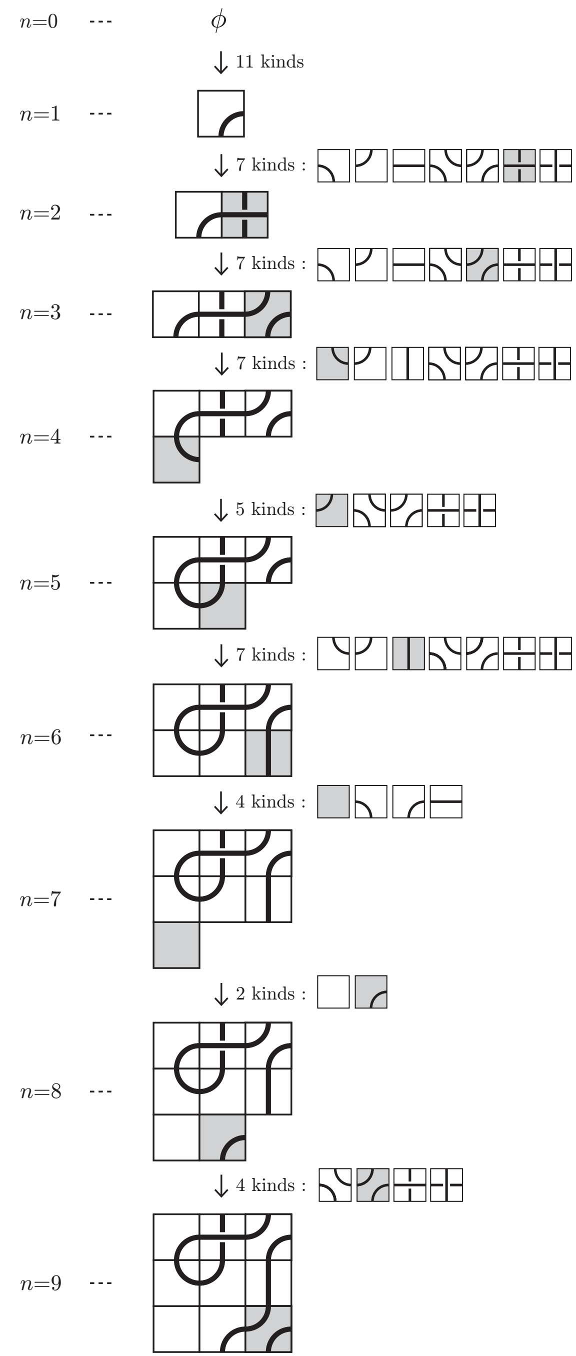
1	2	3
4	5	6
7	8	9

What we're working on – counting configurations

sc-algorithm

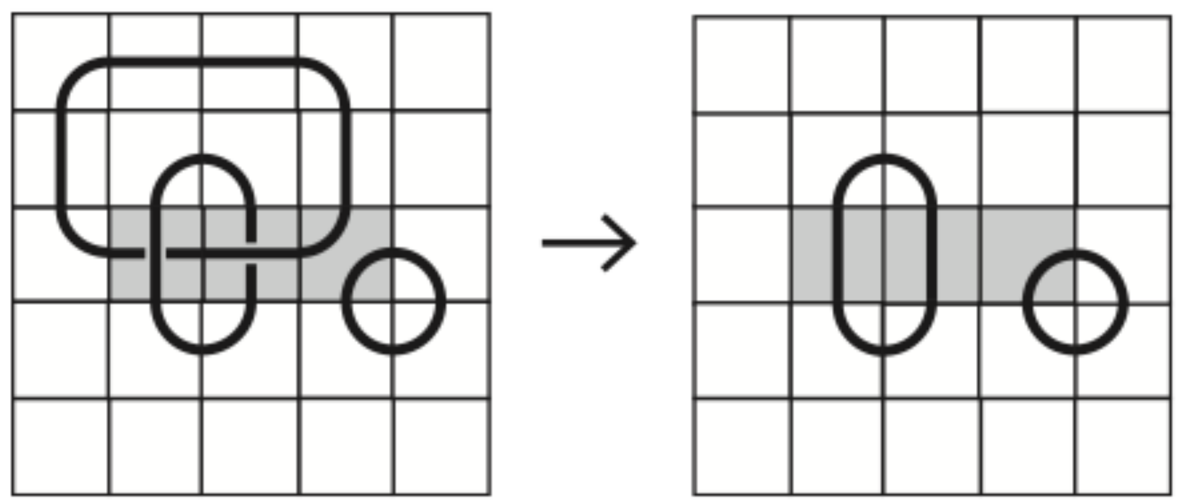
	1	2	3	
	4	5	6	
	7	8	9	

Depth-first search



What else we're working on – counting components

- Given the 2,594 components on a 4x4 board, how many are trefoils, Hopf links, etc.?
- Our technique
  - T_2 initialization
  - sd-algorithm



What else we're working on – counting components – 4x4

Link type	Conformations
Blank board	1
Unknot	1,460
Trefoil	16
Hopf link	56
King Solomon's knot	2
separable 2-component link	860
separable 3-component link	180
separable 4-component link	18
separable 5-component link	1
Total	2,594

What else we're working on – counting components – 5x5

5x5: There are 4,183,954 boards.
0 Component Boards: 1
1 Component Boards: 1440892
2 Component Boards: 1728678
3 Component Boards: 798076
4 Component Boards: 189176
5 Component Boards: 25111
6 Component Boards: 1932
7 Component Boards: 86
8 Component Boards: 2
9 Component Boards: 0

Of the 1 components boards, it follows that
0 Crossing Knot-mosaics: 46684
1 Crossing Knot-mosaics: 182352
2 Crossing Knot-mosaics: 285216
3 Crossing Knot-mosaics: 326336
4 Crossing Knot-mosaics: 289712
5 Crossing Knot-mosaics: 189120
6 Crossing Knot-mosaics: 85376
7 Crossing Knot-mosaics: 28416
8 Crossing Knot-mosaics: 6656
9 Crossing Knot-mosaics: 1024

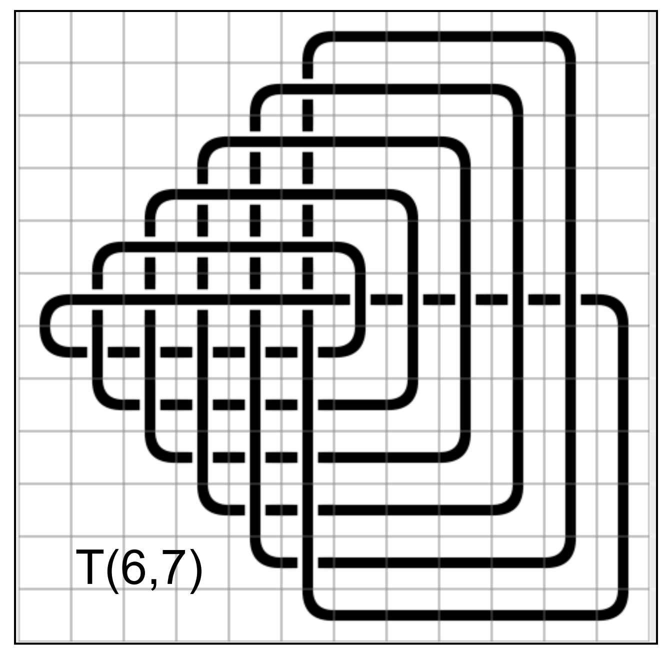
Of the 3-crossing knot mosaics, there are 14248 trefoils.

What we'd like to know

- What are the right counting questions?
- Mosaic number for all small knots:

9 and 10 crossings (49 + 165)

- $m(T_{(p,p+1)})=2p$?



Thanks

- Elizabeth and Laura
- Colin
- Sam and Lou
- My students