Knot Mosaics: Results and Open Questions

Lew Ludwig

Denison University

Our program

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- Review of knot mosaics

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- Some recent results in knot mosaics

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- Some recent results in knot mosaics
  - Arc presentation
  - Grid diagram

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  - Grid diagram
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- Determine the mosaic number for an infinite family of knots
- Conclude with open questions

What are Knot Mosaics?

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- Lomonaco and Kauffman (2008)

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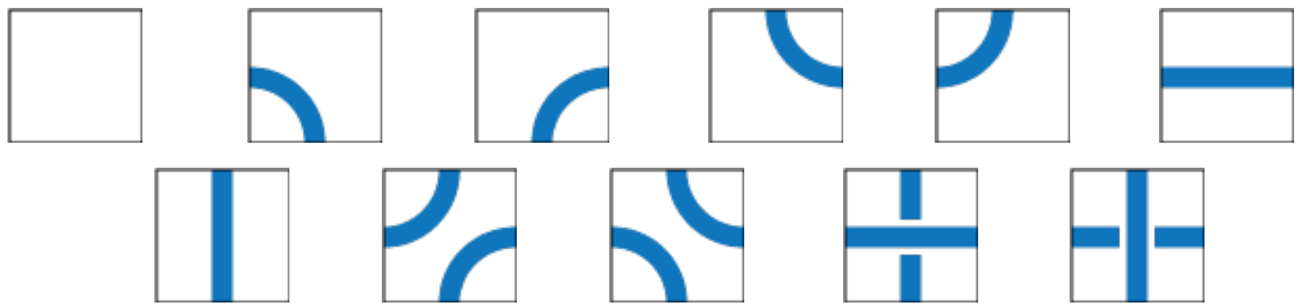

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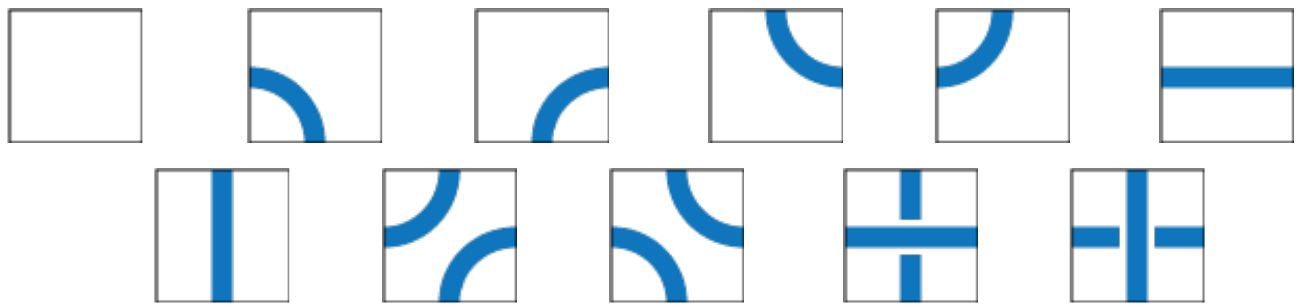
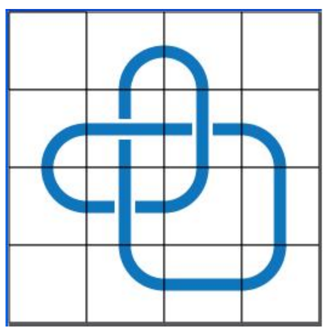




Trefoil

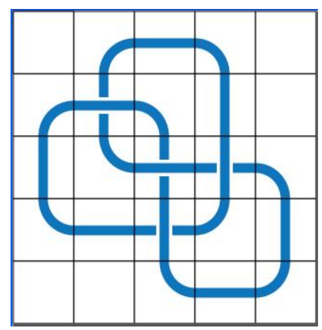
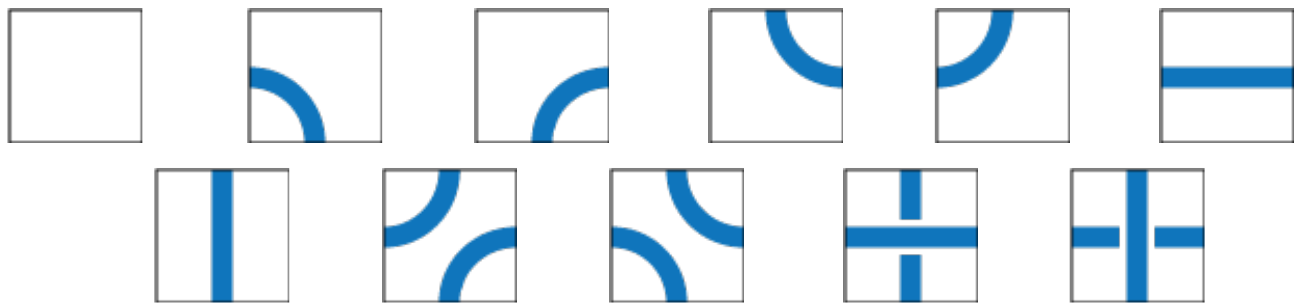
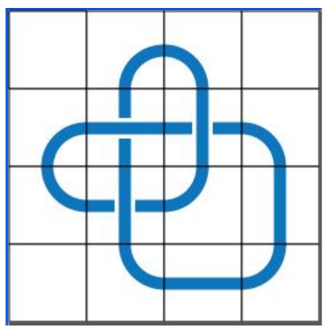


Figure 8

What are Knot Mosaics?

- Lomonaco and Kauffman (2008)





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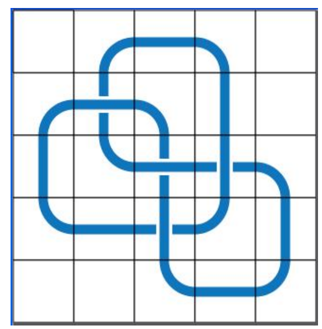

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- Kuriya (2008), Shebab (2012) tame knot theory and mosaic knot theory are equivalent

Some terminology

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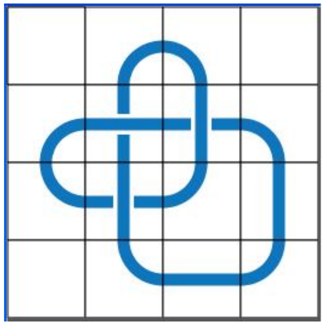

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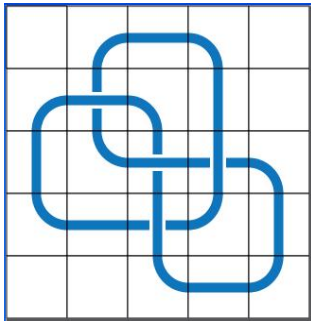

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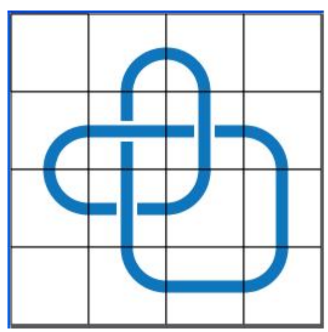
The minimal size mosaic board that a knot will fit on



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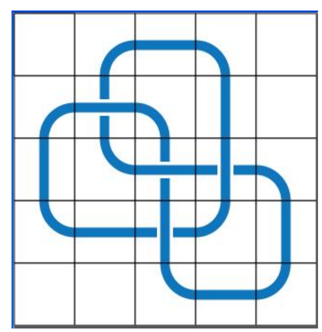

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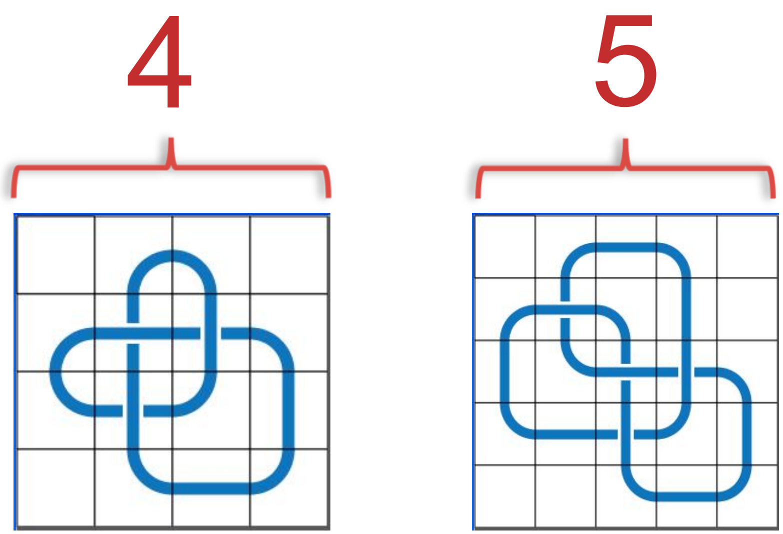

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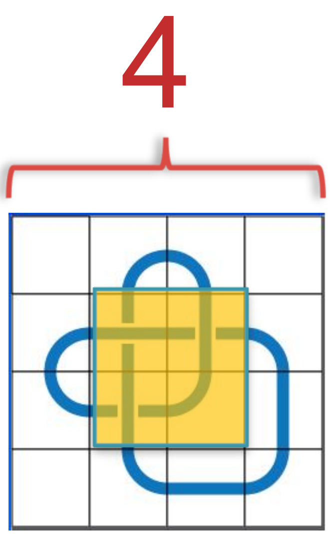

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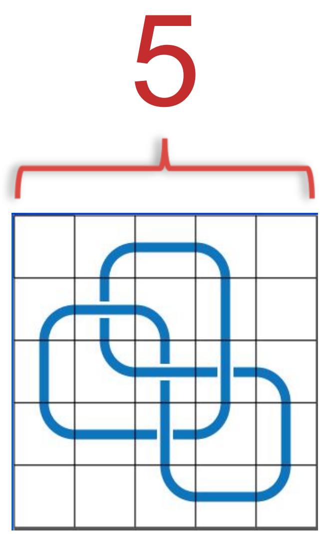

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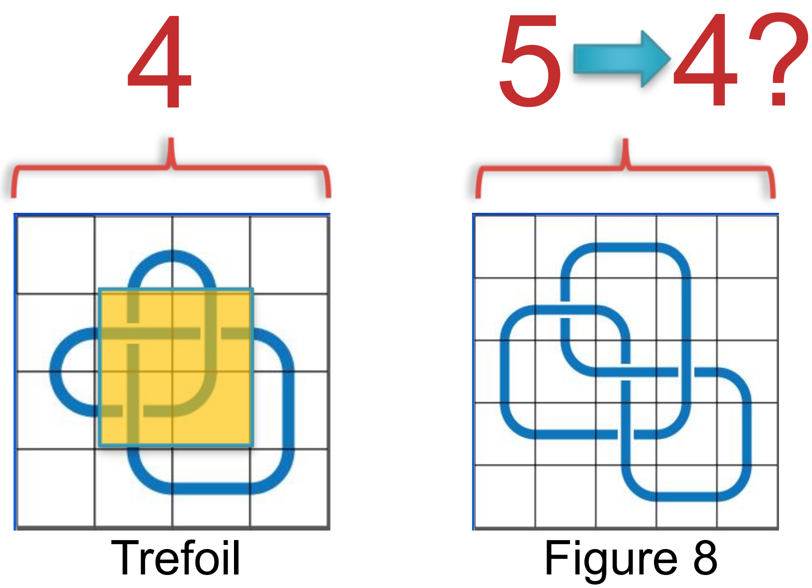

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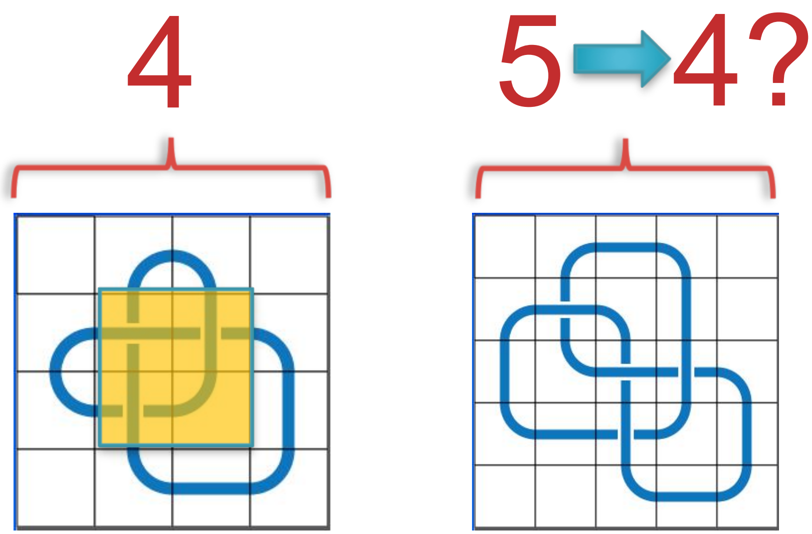

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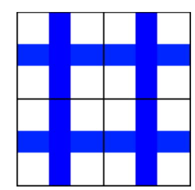

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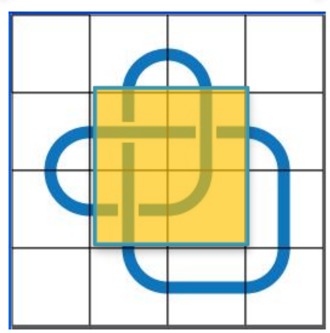
$m(3_1)=4$ $m(4_1)=5$



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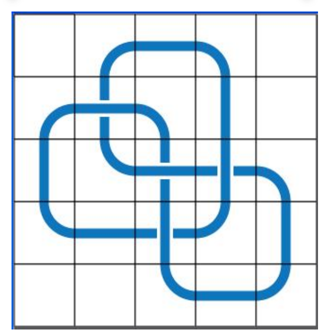

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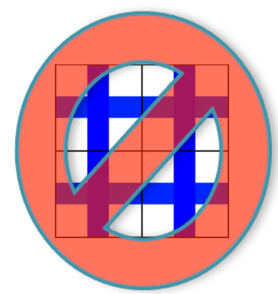

Question – Lomonaco and Kauffman

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- Is the mosaic number, $m(K)$, related to the crossing number, $c(K)$, of a knot K?

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Some useful tools

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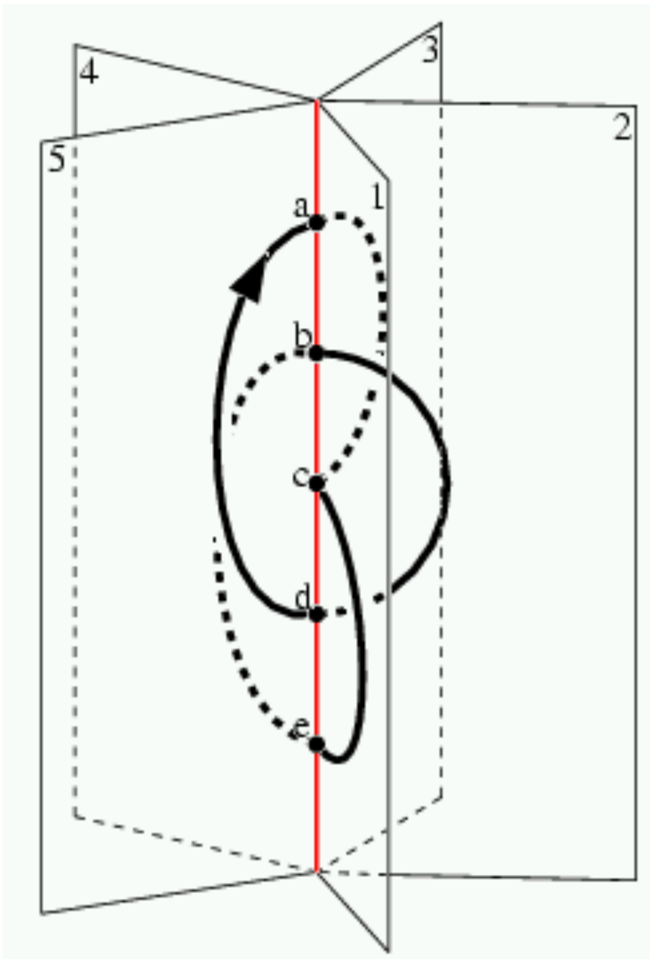

Arc presentation

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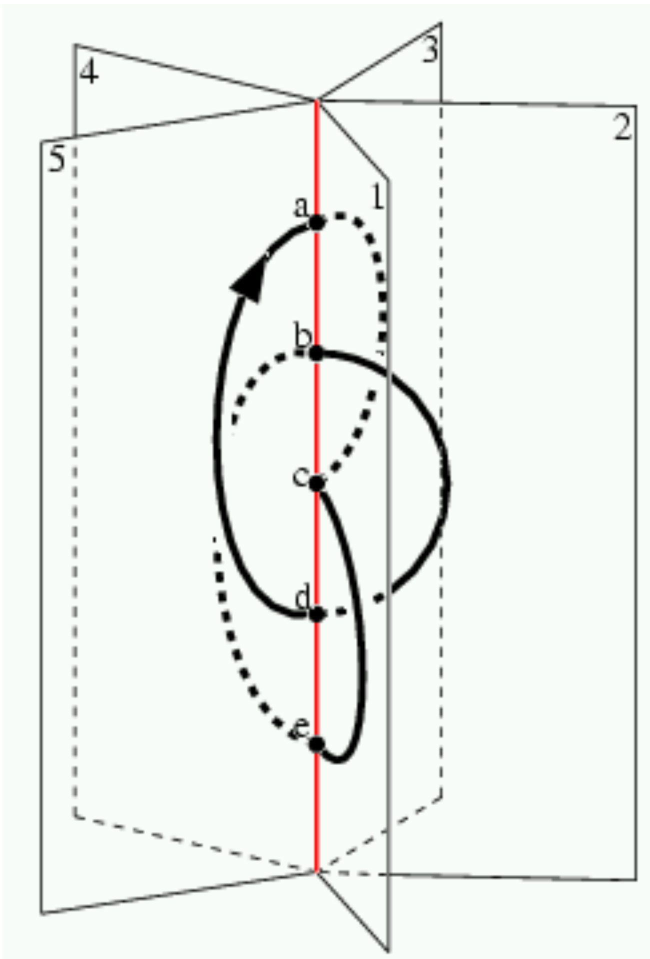

Arc presentation

- z-axis is binding

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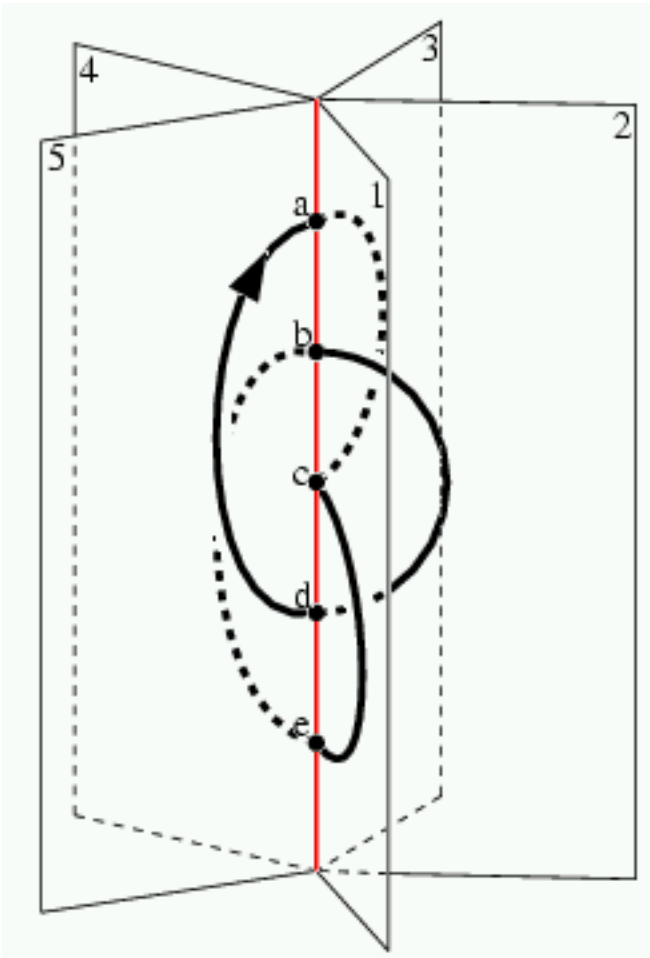

Arc presentation
- z-axis is binding
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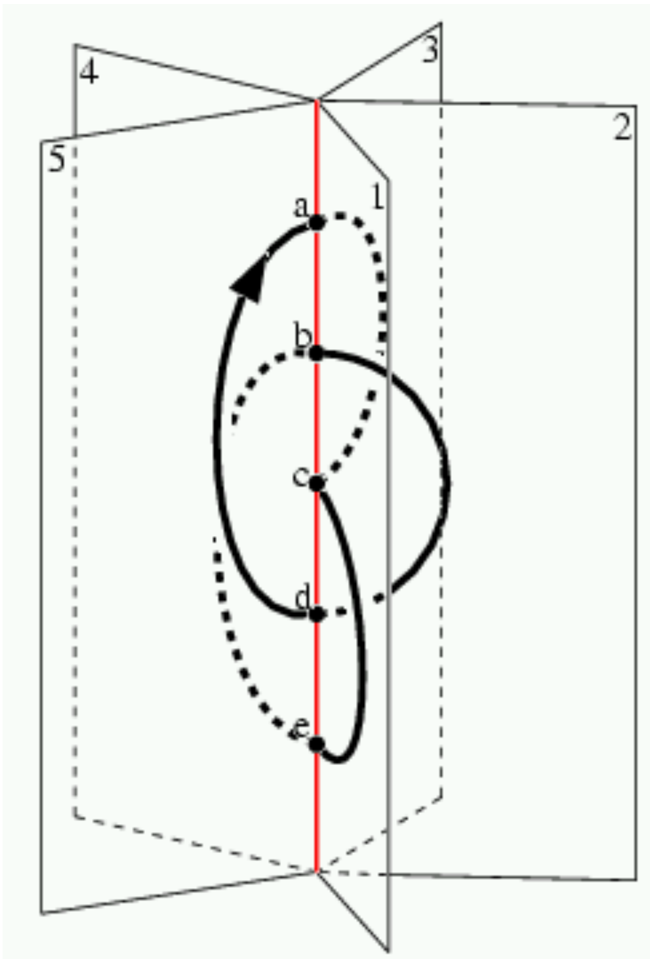

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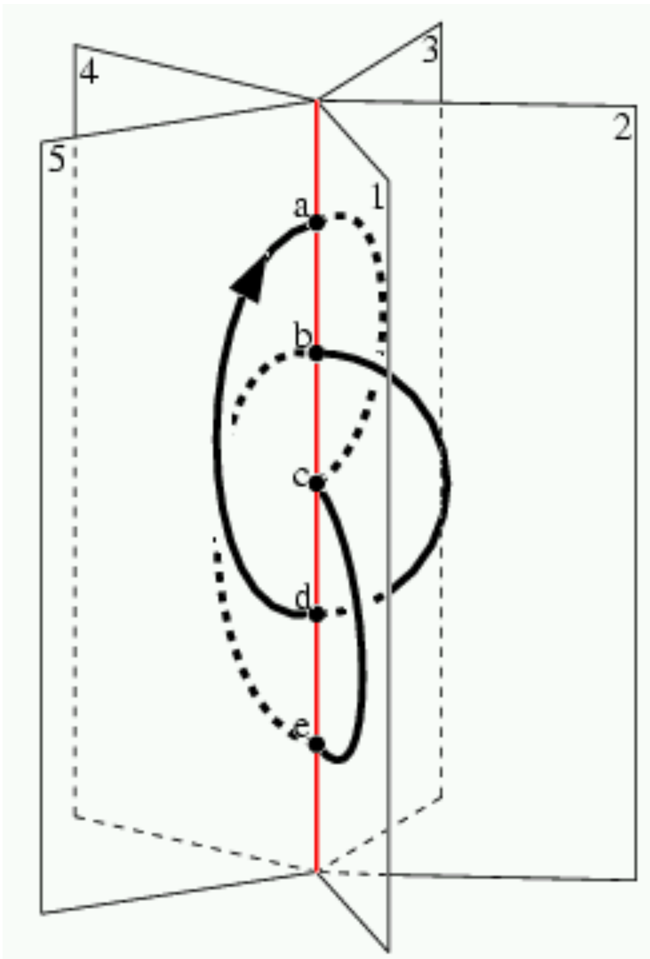

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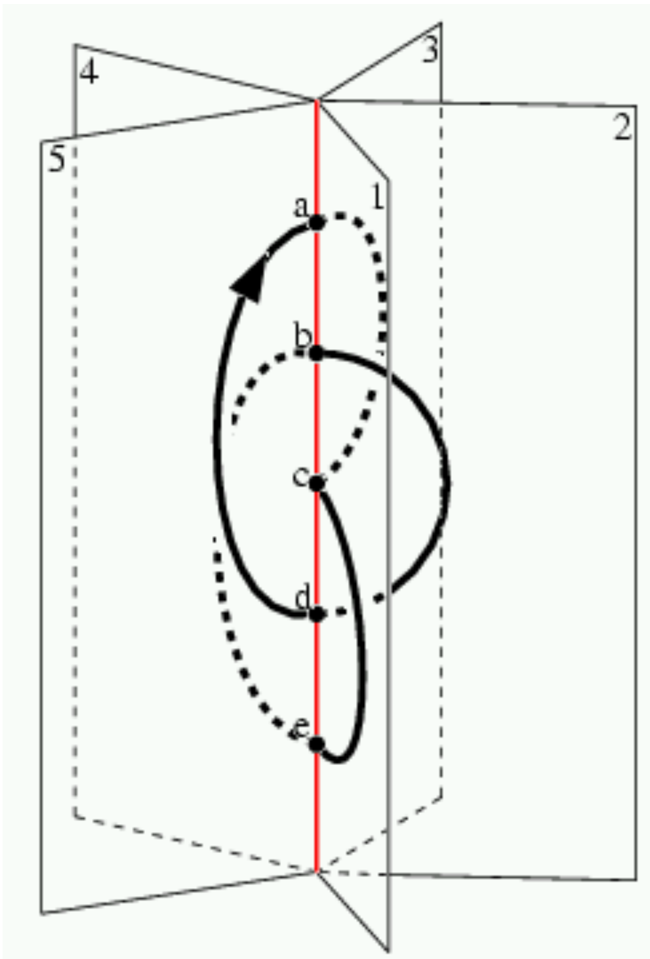

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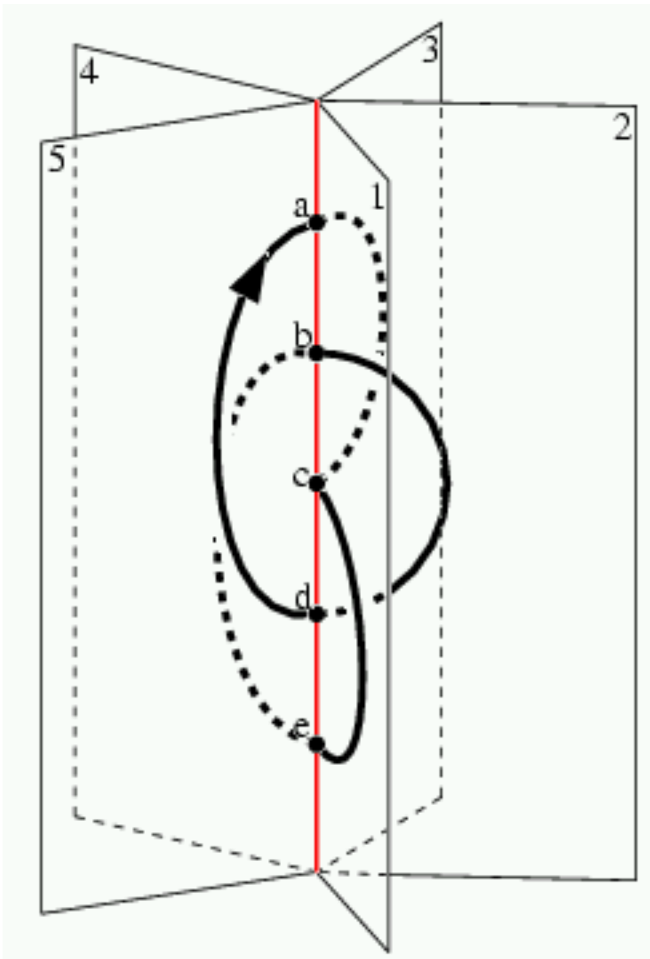
Arc index, $\alpha(K)$, minimum number of pages required

What is $\alpha(K)$?

- Brunn 1897
- Birman & Menasco 1990s
- Cromwell 1990s
- Recently Heegaard Floer Homology



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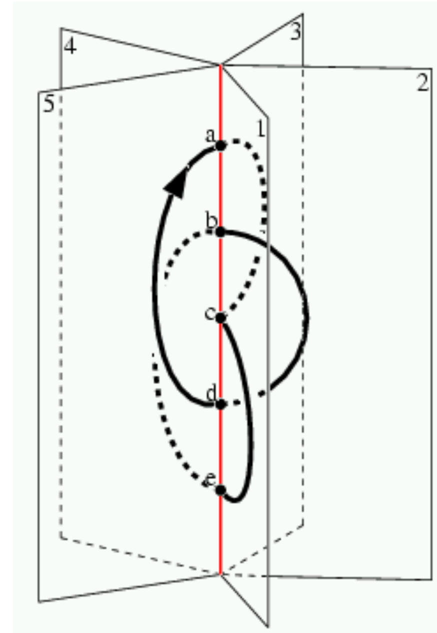

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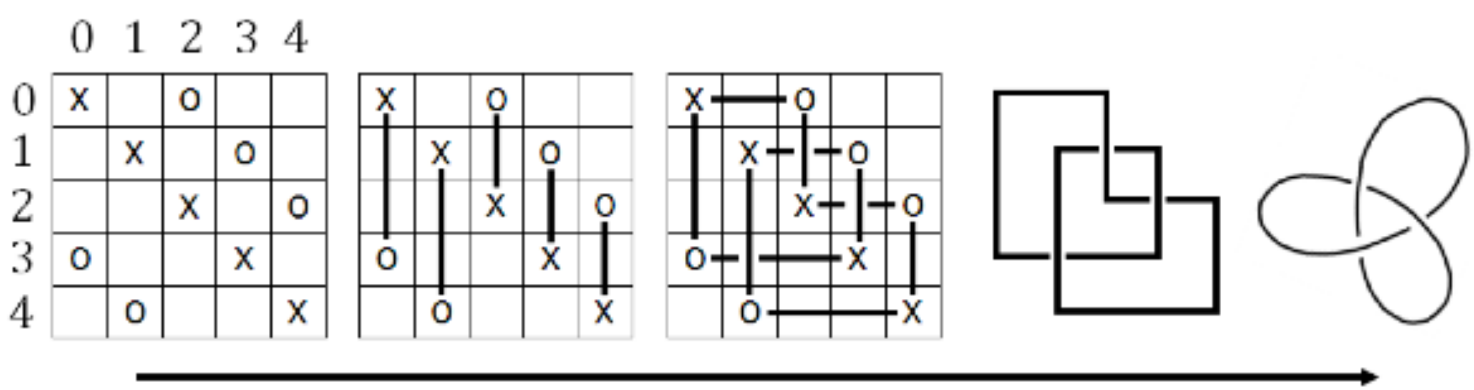

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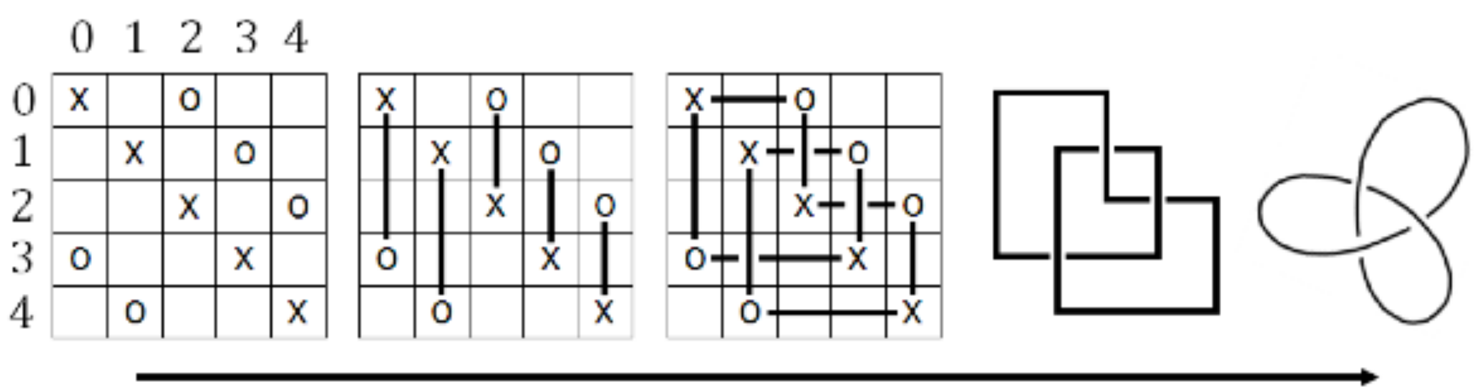

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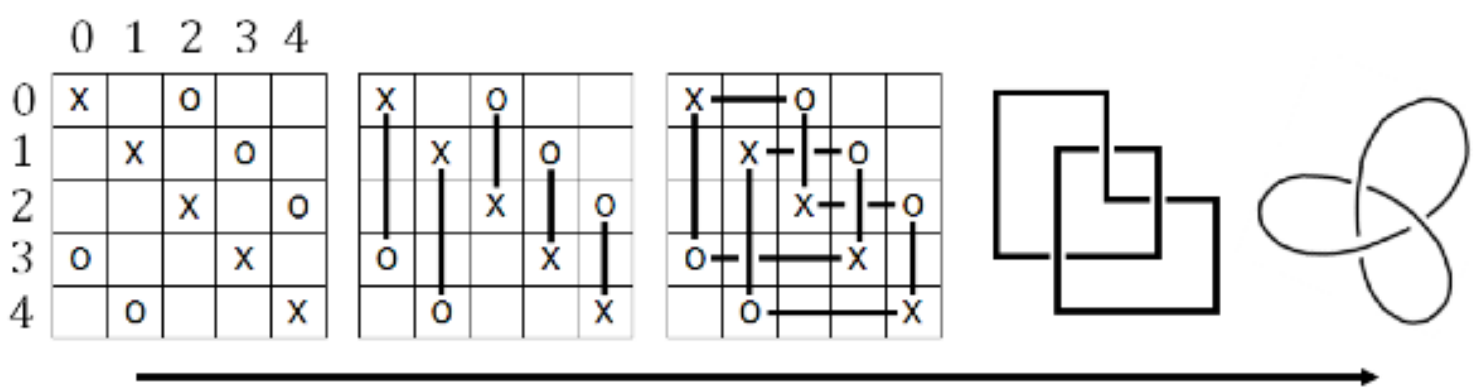

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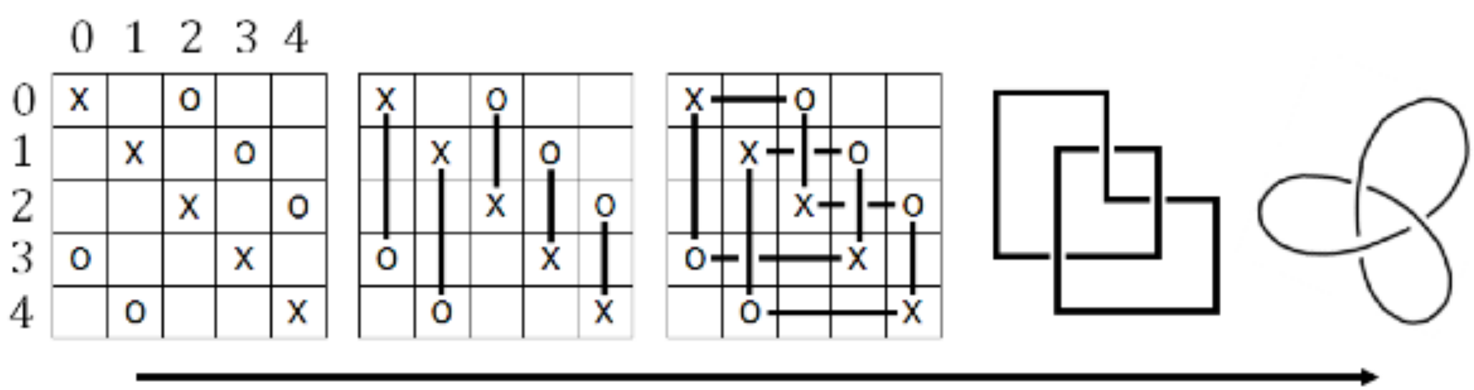

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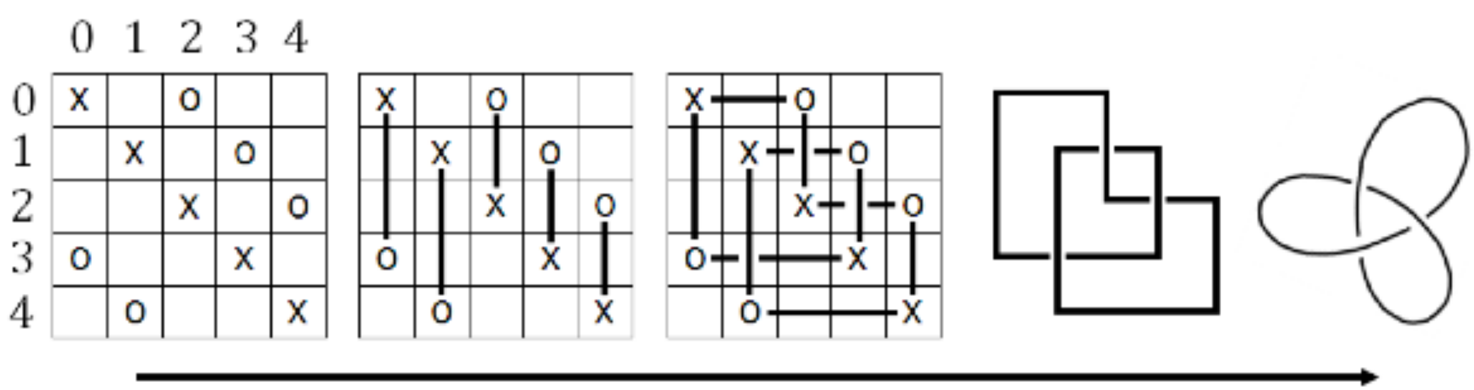

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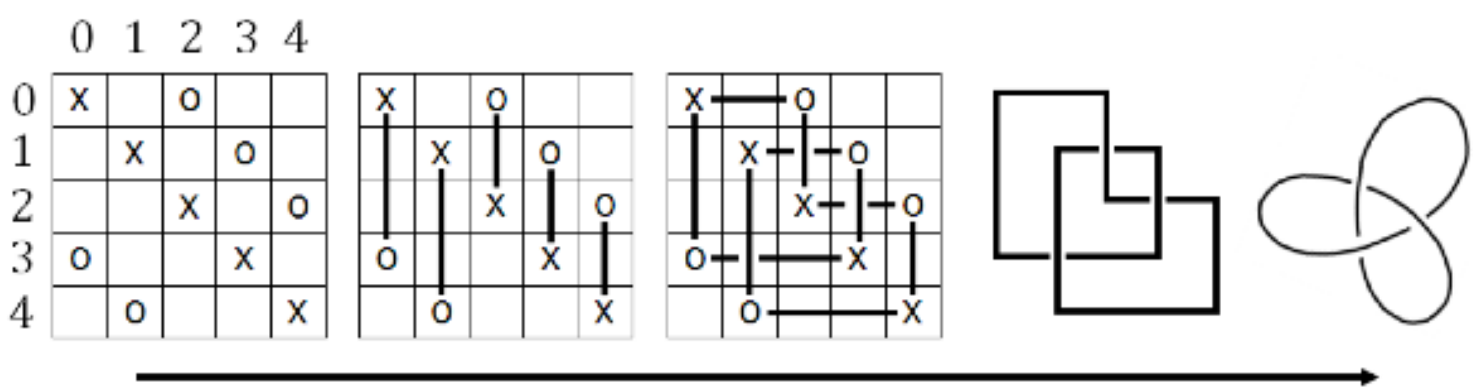
- Grid Index, G(*K*), min # vertical segments



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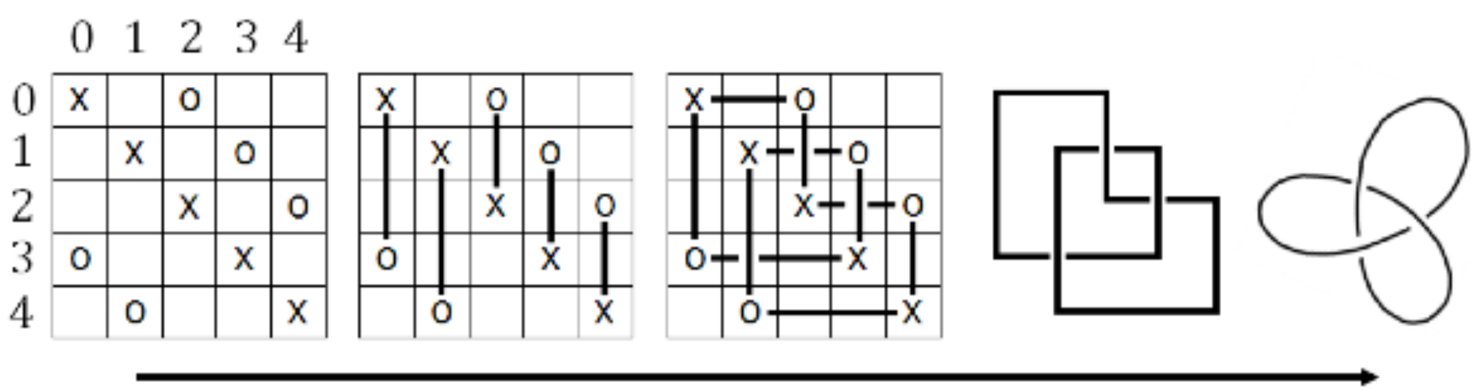

★Natural connection to knot mosaics★

Cromwell grid moves (Dynnikov)

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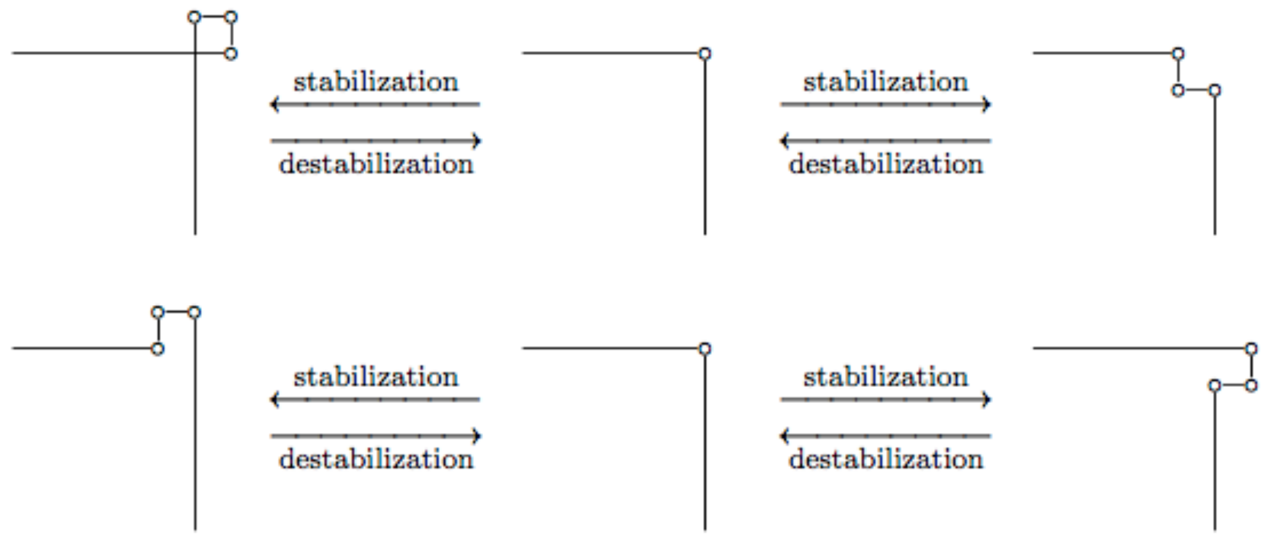
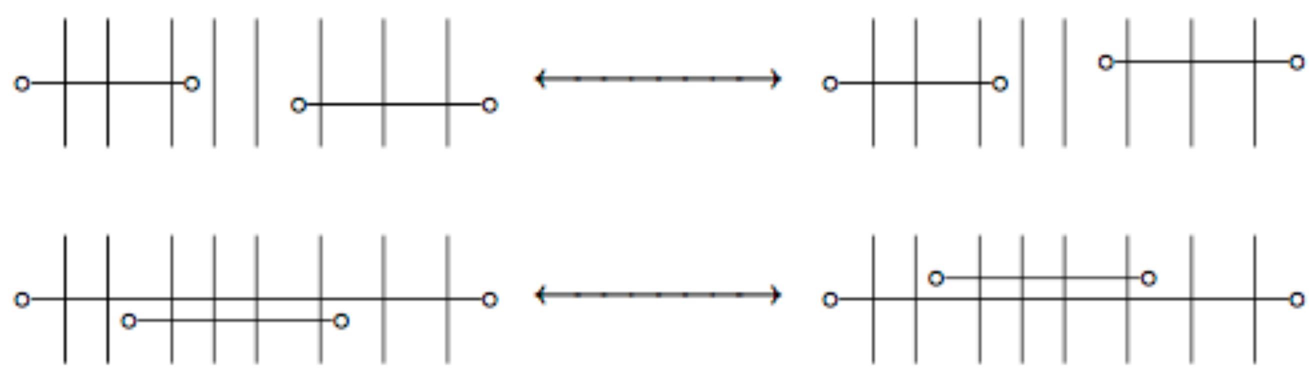


Cromwell grid moves (Dynnikov)

- Stabilization and destabilization
- Interchanging neighboring edges if their pairs of endpoints do not interleave



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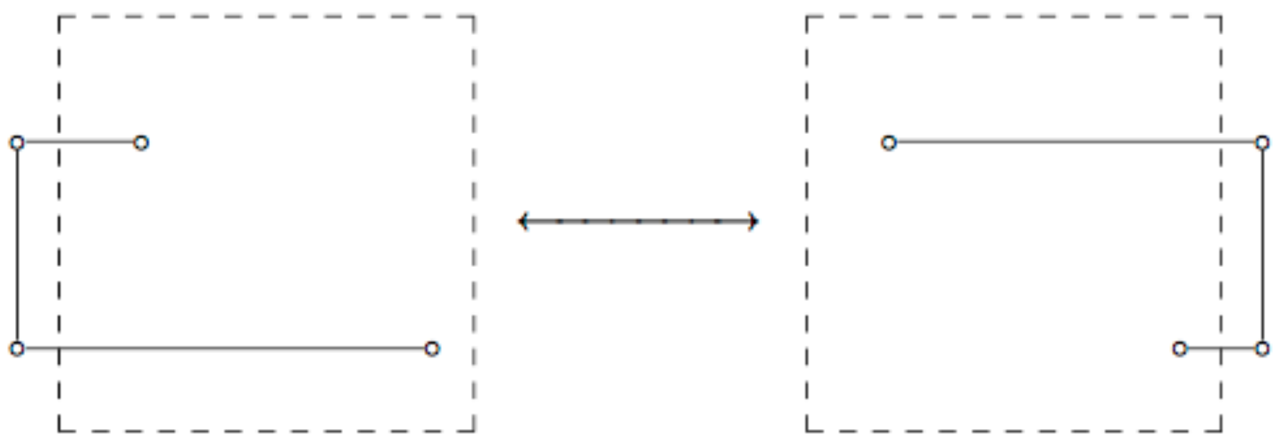

Cromwell grid moves (Dynnikov)

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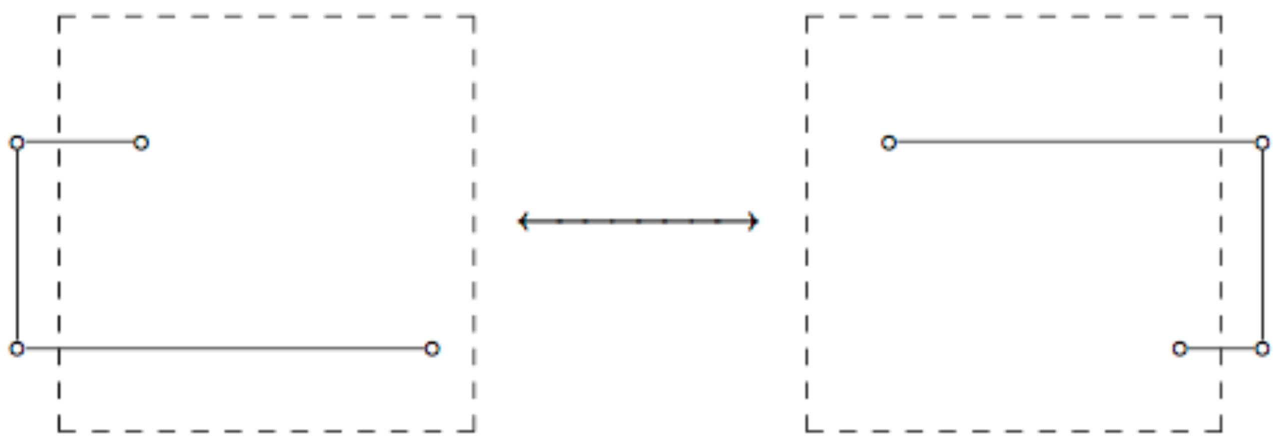
- ★Cyclic permutation of vertical (horizontal) edges – do not change G(K)



Keep goal in mind…

Knot *K*: $m(K) \leq c(K)+1$

Knot *K*, prime non-alternating: $m(K) \leq c(K)-1$

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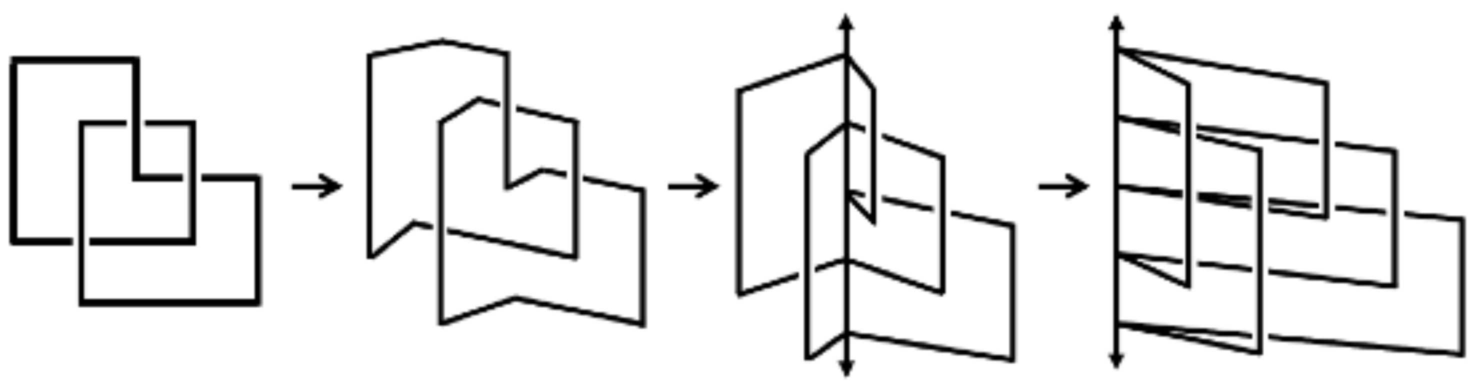
One more connection: grid diagram and arc presentation



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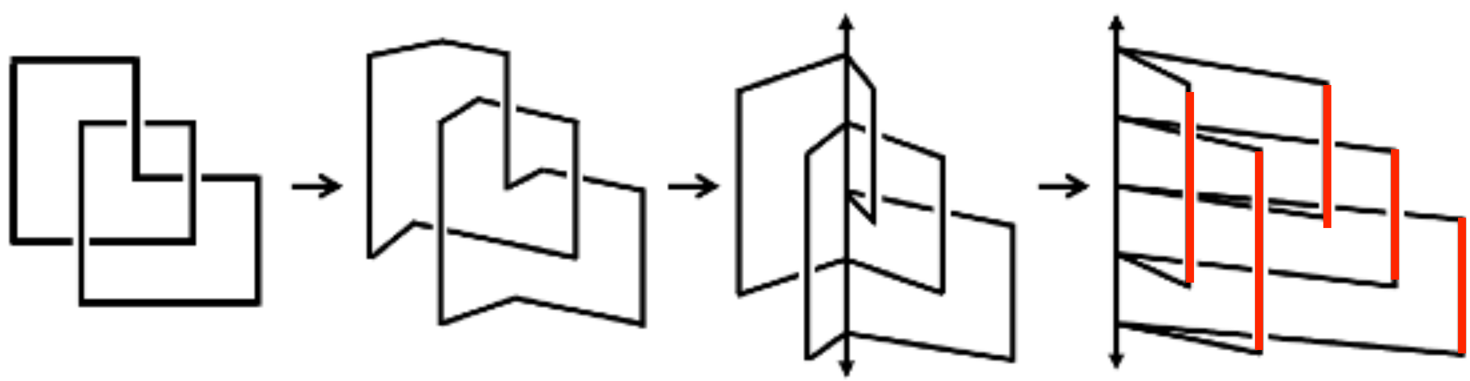

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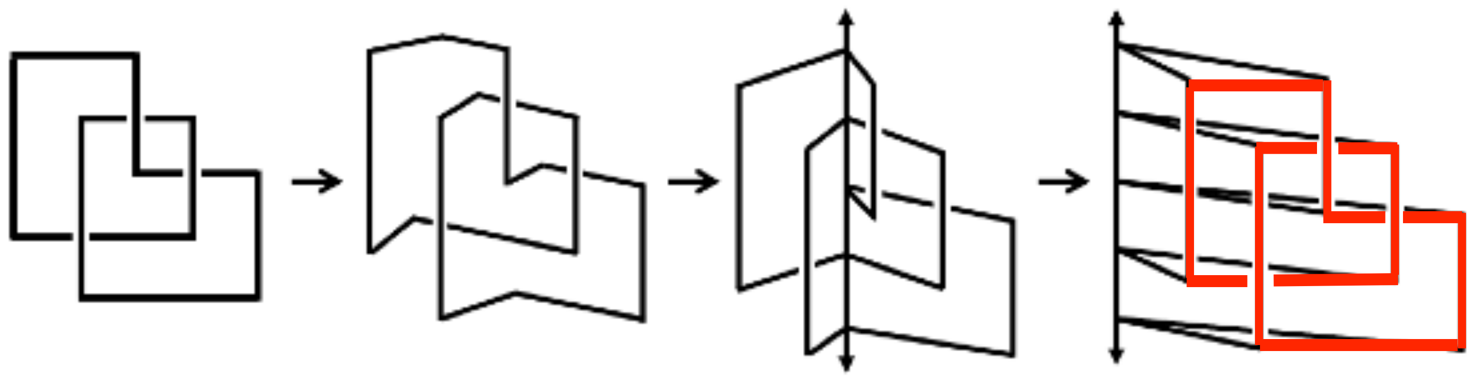

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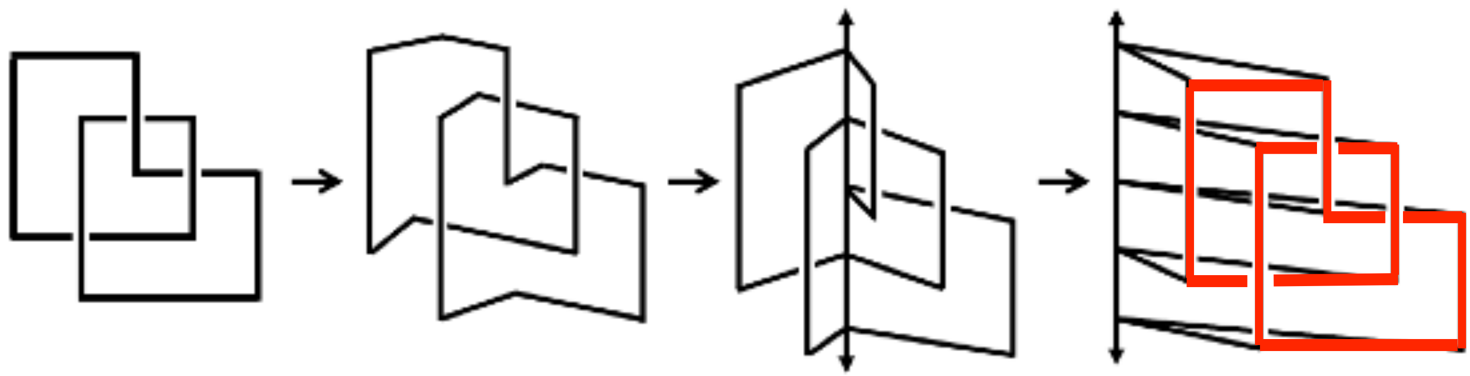

★Arc index (K)= Grid Index (K)★

Now for the proof: $m(K) \leq c(K)+1$

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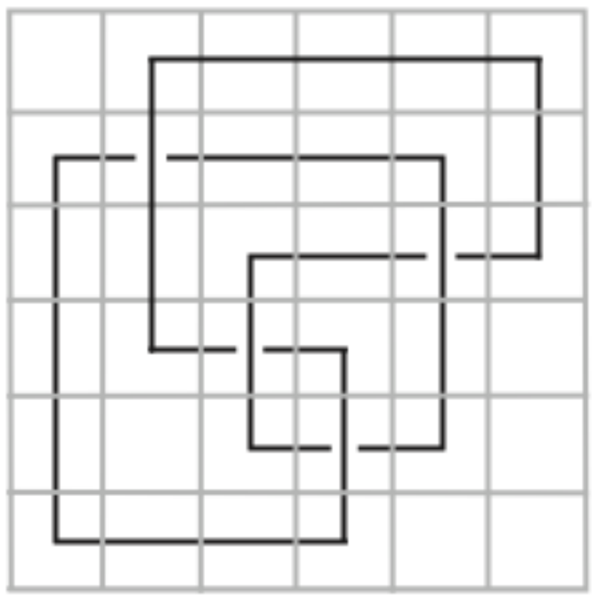

Fig. 1

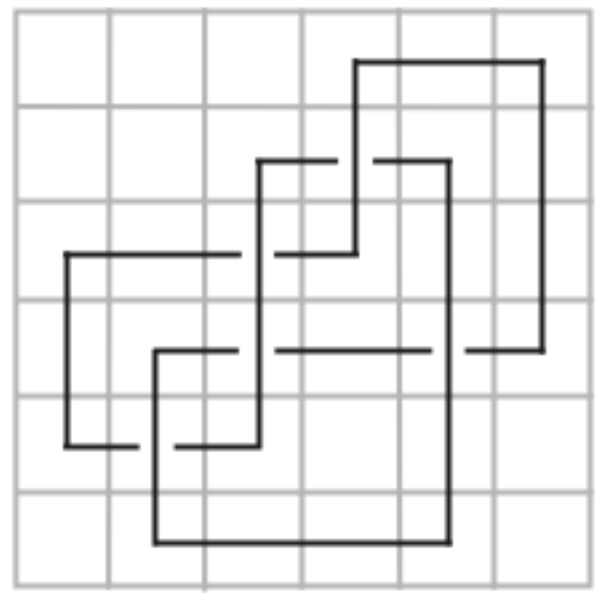

Fig. 2

Now for the proof: $m(K) \leq c(K)+1$

Notice the
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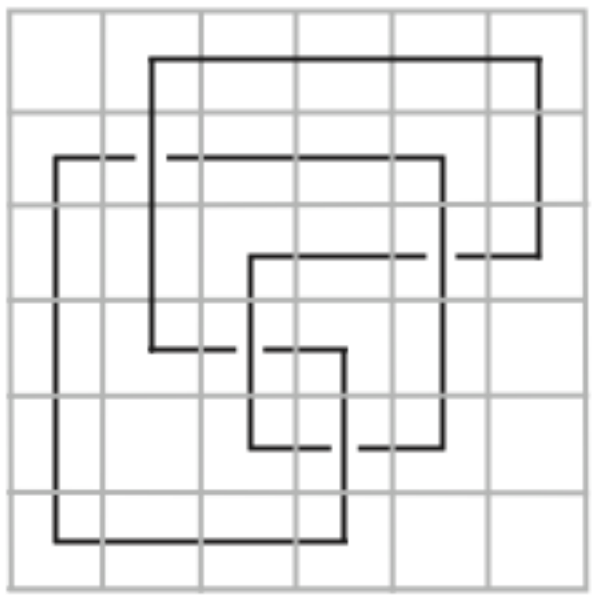

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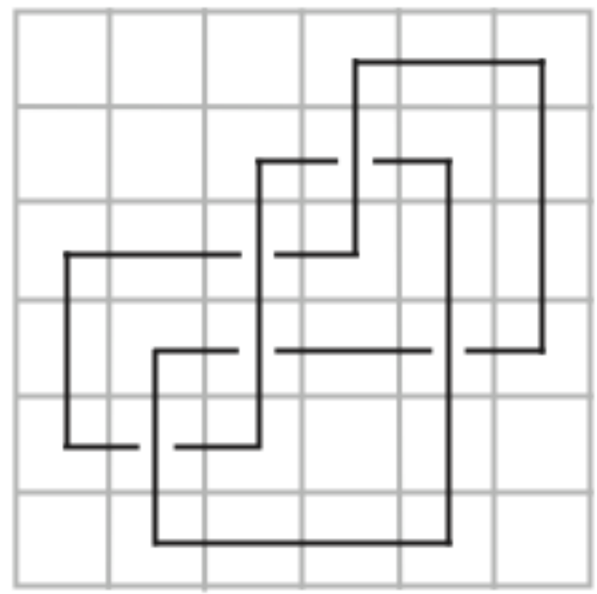

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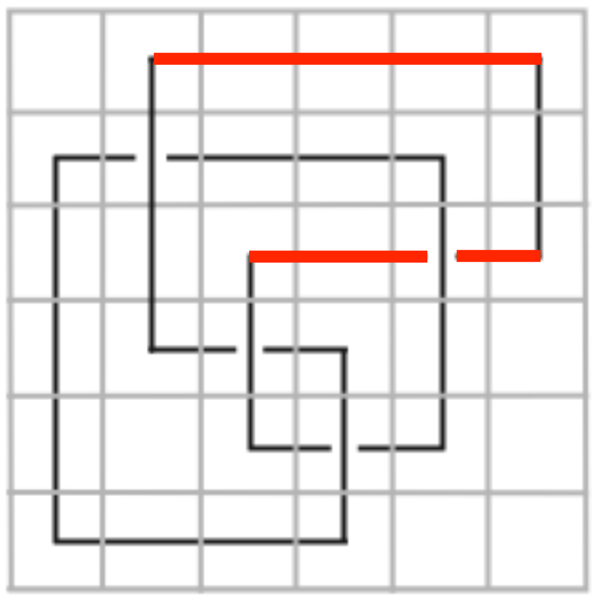

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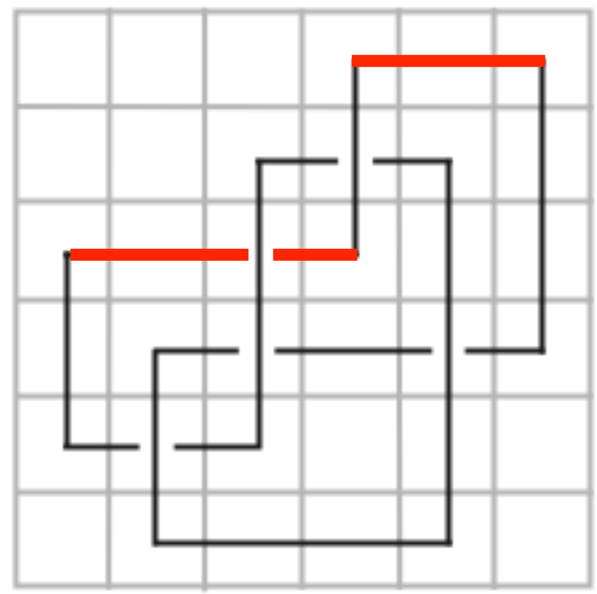

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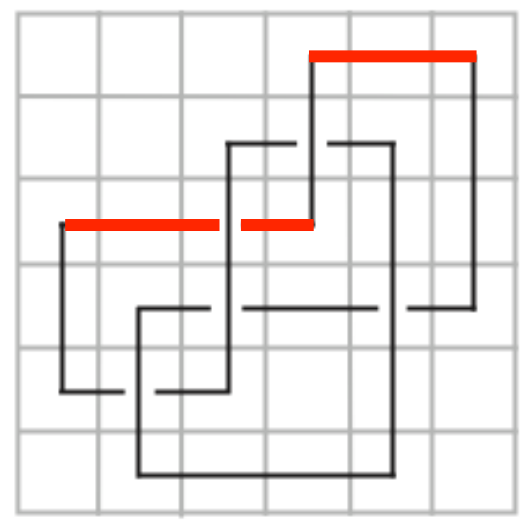

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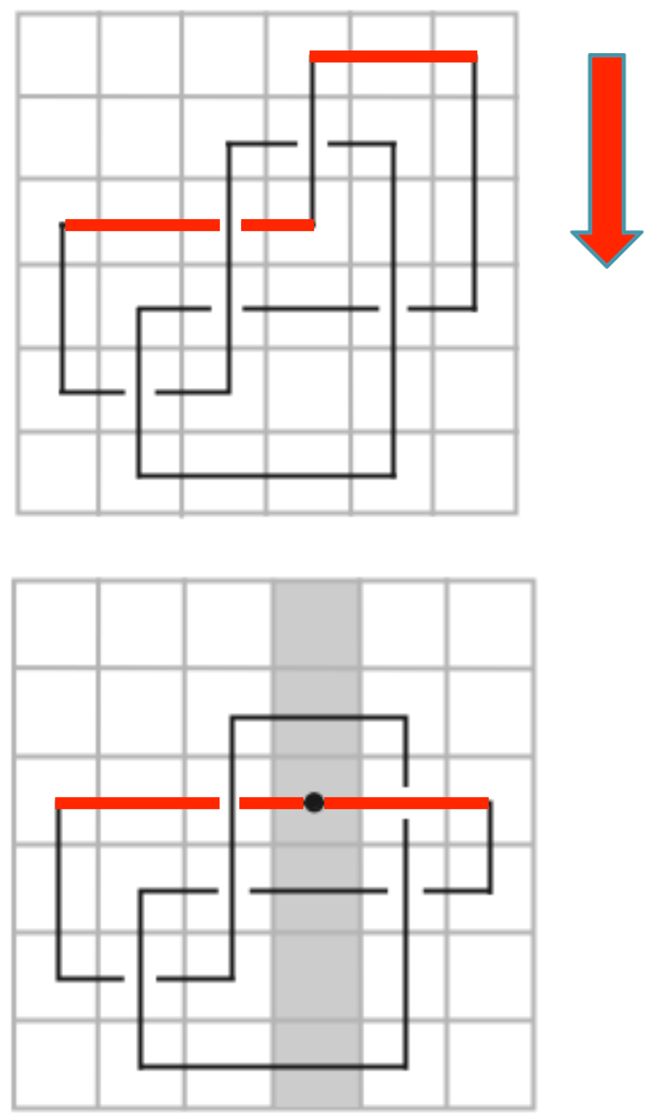

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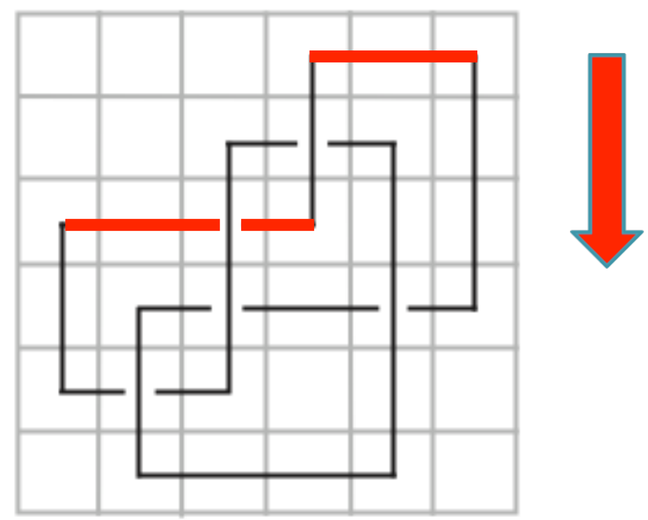
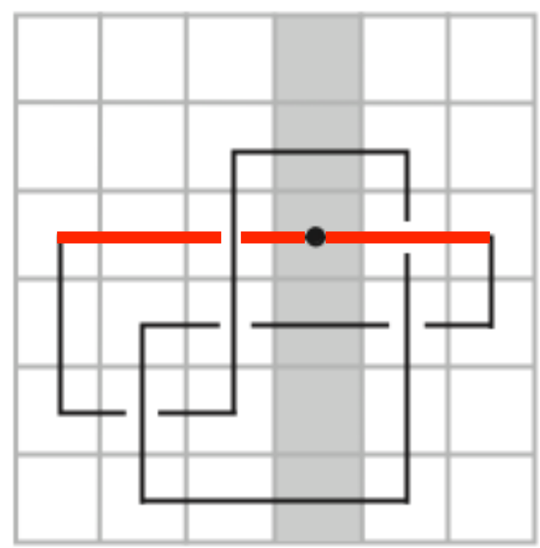


Reduced the mosaic size by 1



Now for the proof: $m(K) \leq c(K)+1$

Notice the
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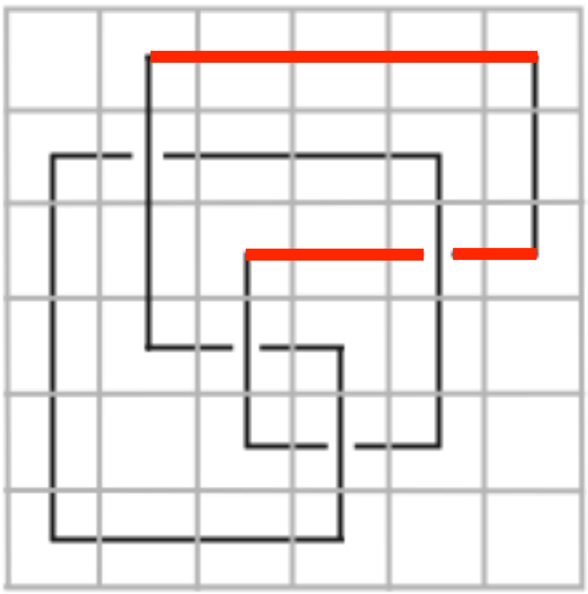

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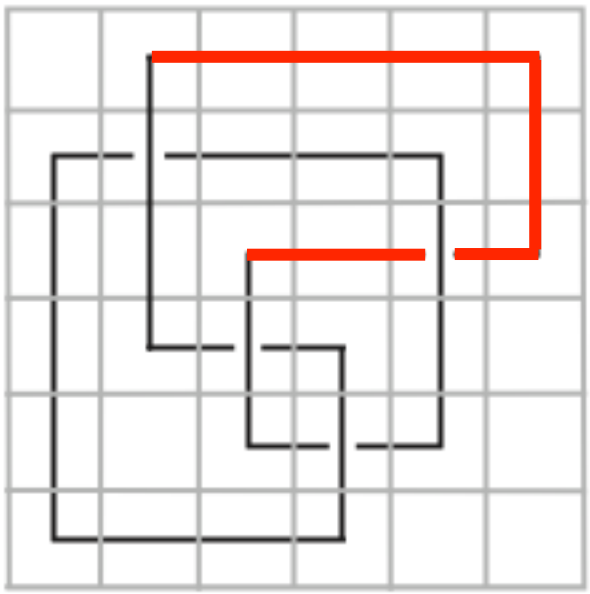

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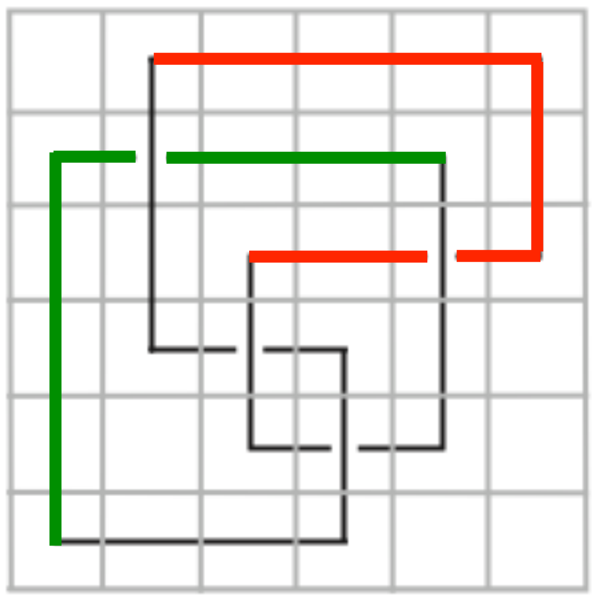

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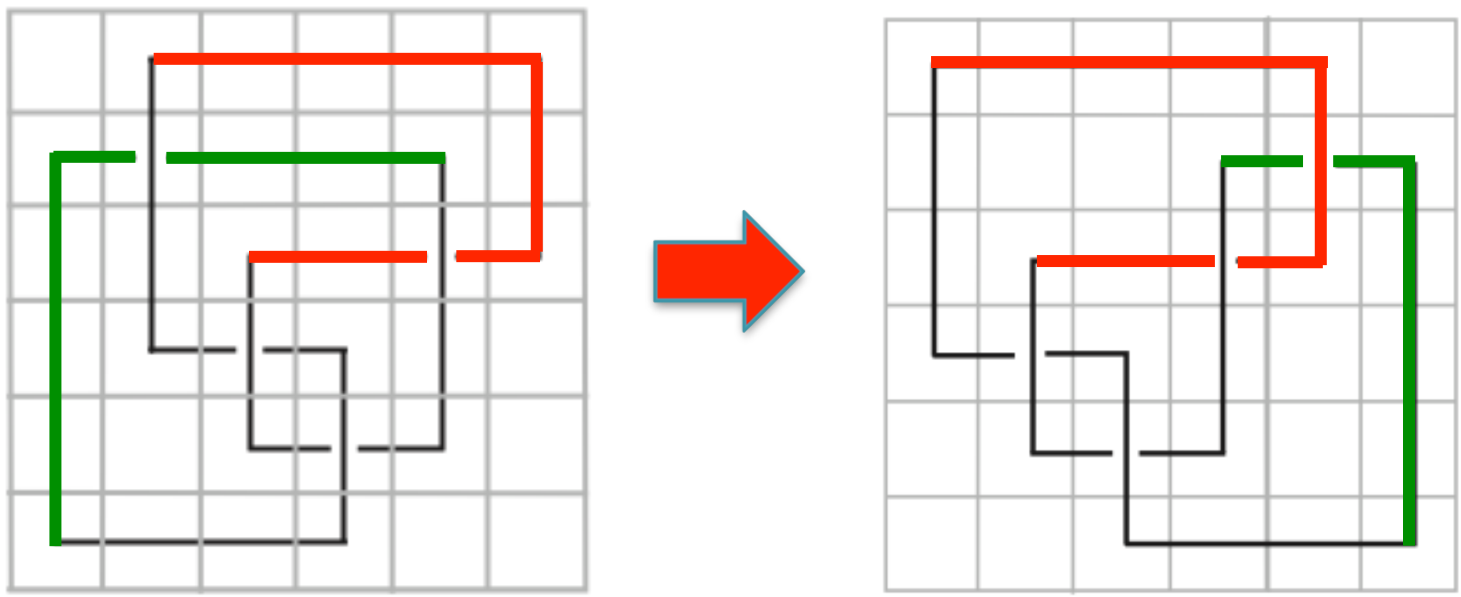

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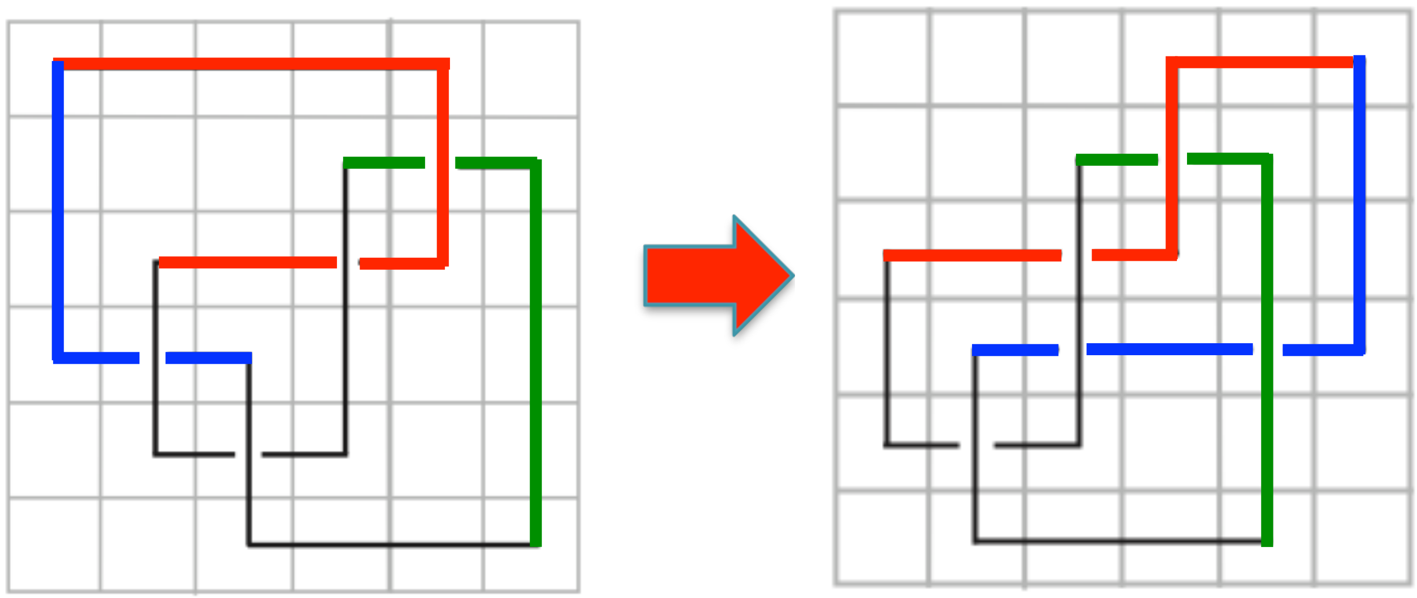

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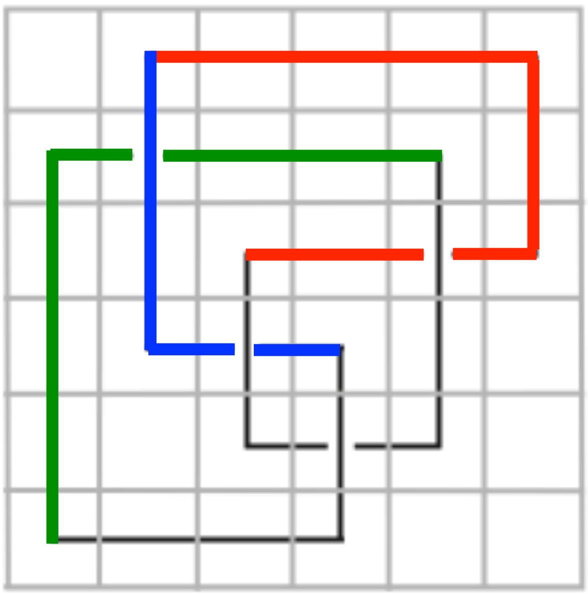

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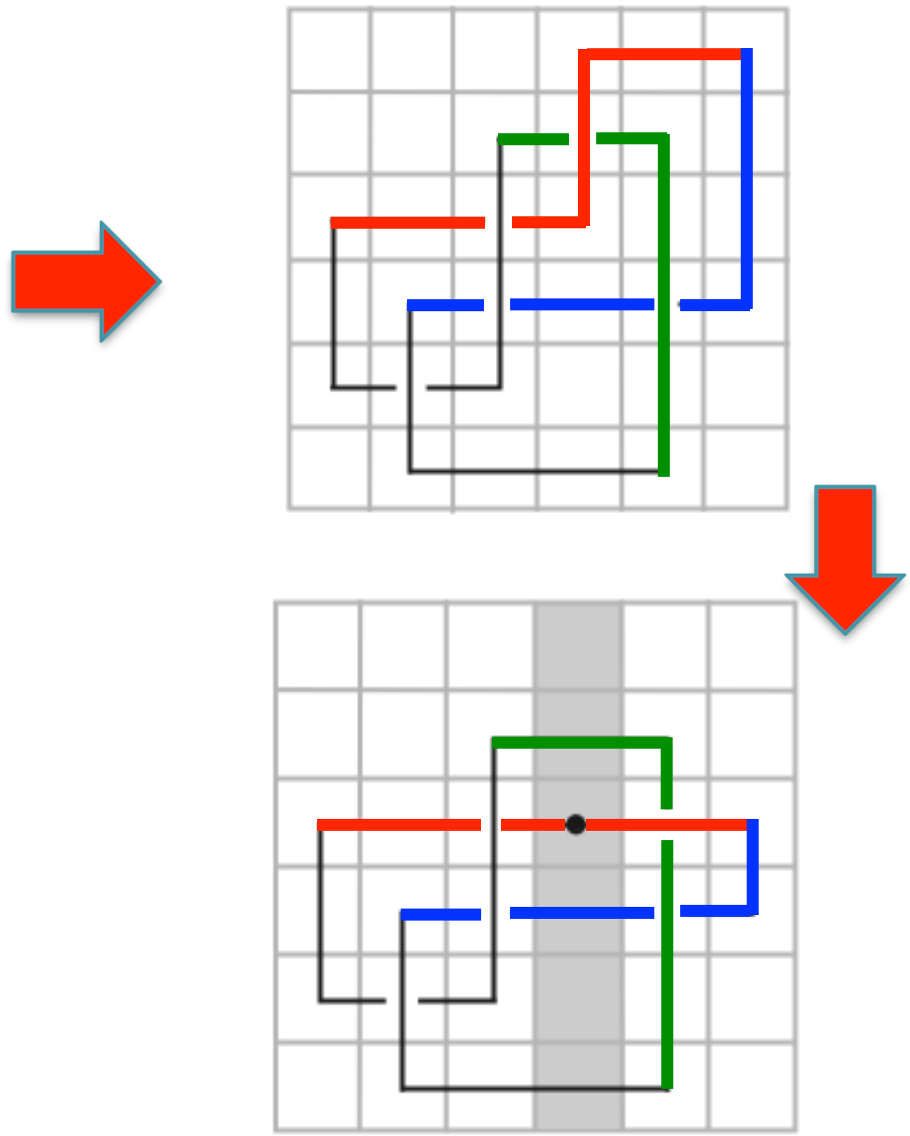
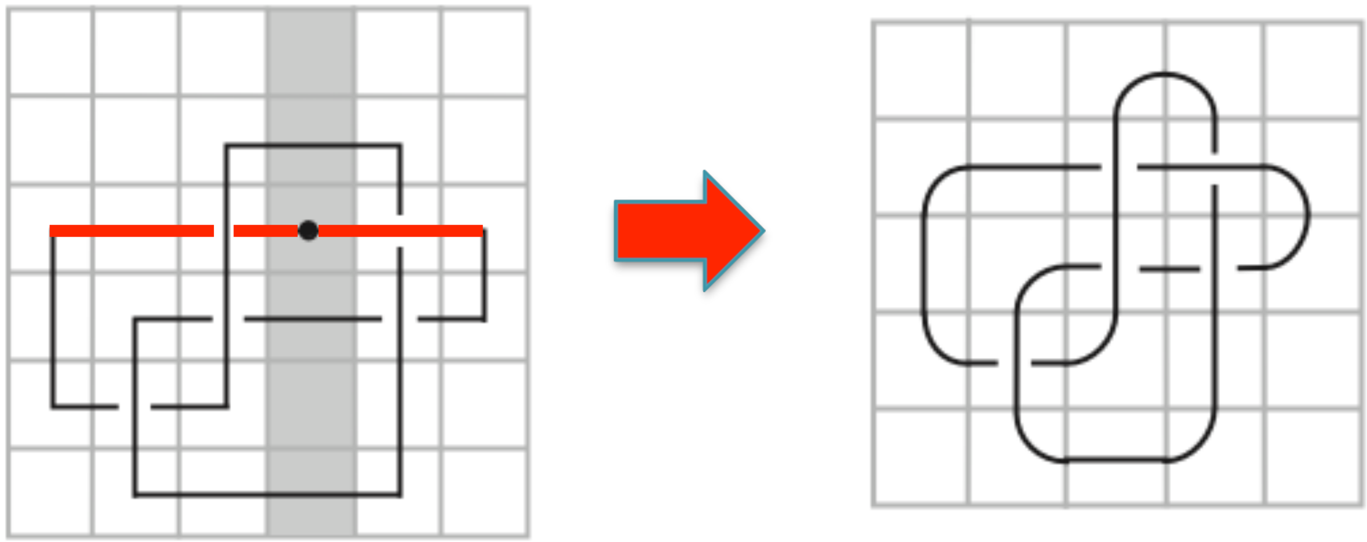


In either case…



So $m(K) = \alpha(K) - 1$

$\leq (c(K) + 2) - 1$ Bae & Park

$= c(K) + 1$

and $m(K) \leq c(K) - 1$ if non-alt prime Jin & Park

A bound on the mosaic number of an infinite family of knots

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- (L. & Wu, 2012)

$$m(T_{(p,p+1)}) \leq 2p$$

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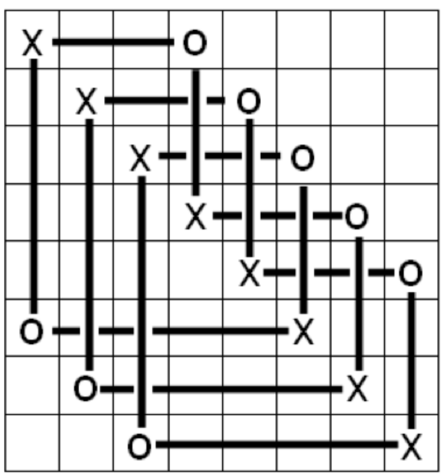

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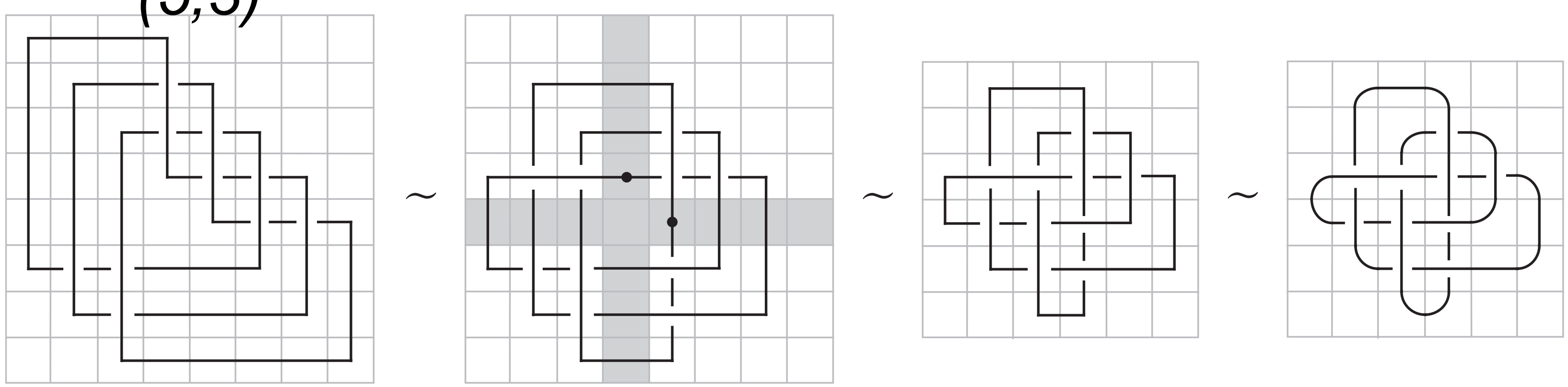

The mosaic number of an infinite family of knots

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The Question – Adams 2009

- Is there an infinite family of knots whose mosaic number is realized only when the crossing number is not?

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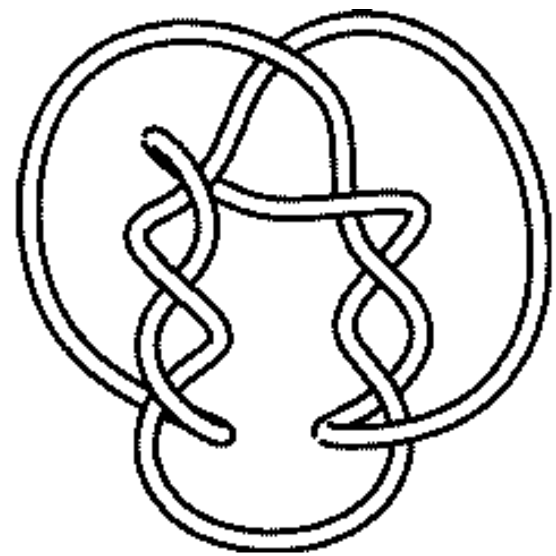

- Why is this *interesting*?
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- Bernhard 1994, generalized

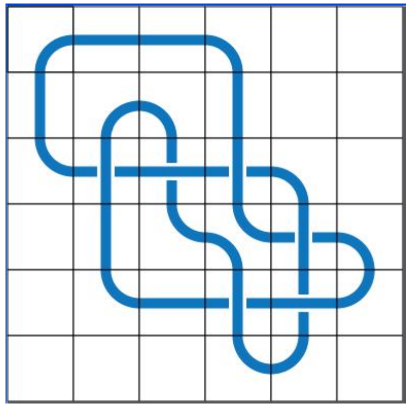
Nakanishi 1983 result – infinite family of knots whose

unknotting number is realized when the crossing number is NOT!

The mosaic number of an infinite family of knots

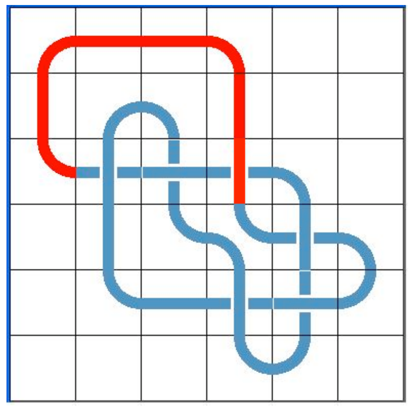
- Is there an infinite family of knots whose mosaic number is realized only when the crossing number is NOT?

Our Construction



Knot 6_1
Number of Crossings: 6
Mosaic Size: 6

Our Construction (Jacob Shapiro, '10)



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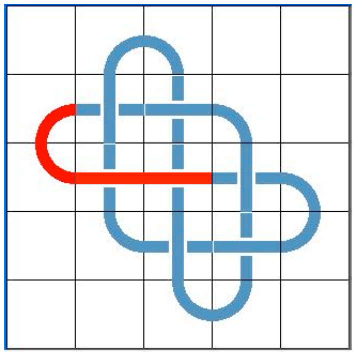
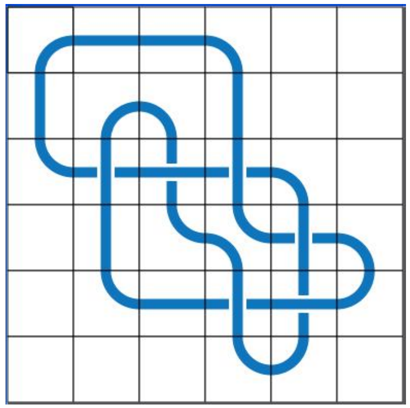


Number of Crossings: 7
Mosaic Size: 5

Our Construction



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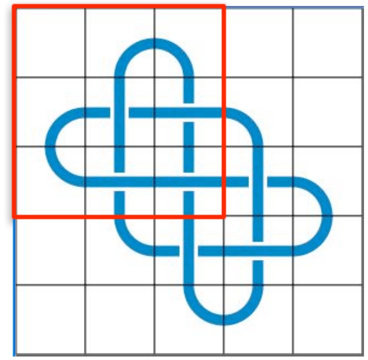

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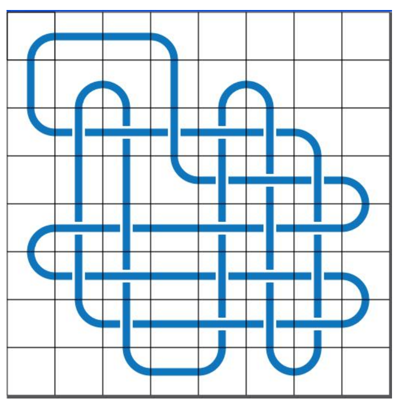
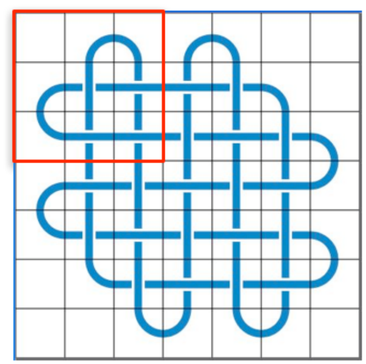


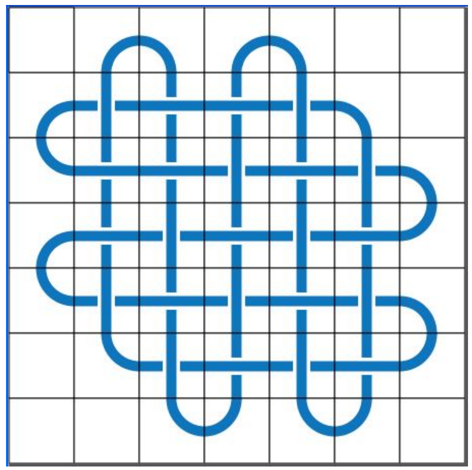
Number of Crossings: 22
Mosaic Size: 8



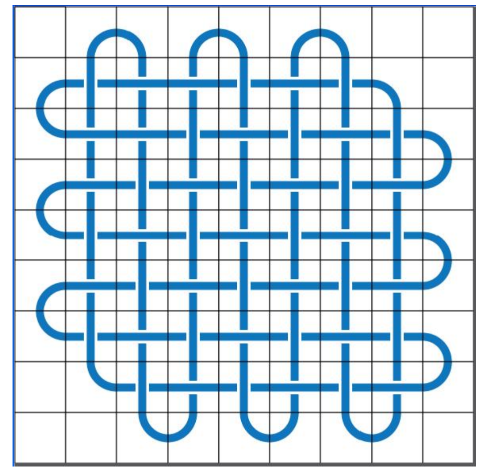
Number of Crossings: 23
Mosaic Size: 7

What is our Game Plan?

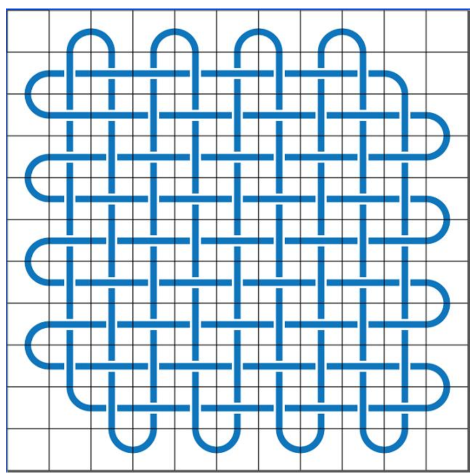
- L_7



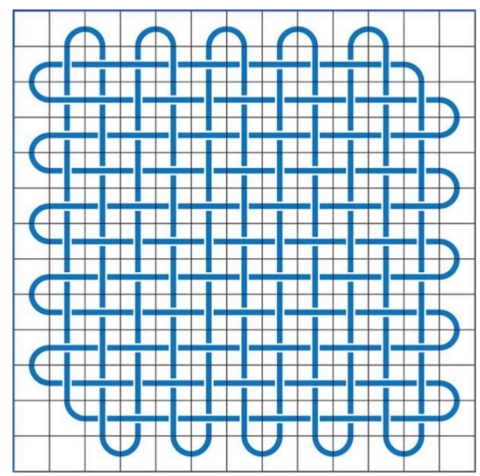
- L_9



- L_{11}



- L_{13}



Claim: L_{2n+1} is the family we seek,

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1. Must compute crossing number for this family.

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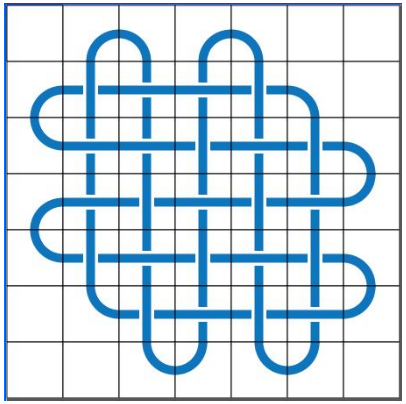
1. Must compute crossing number for this family.
2. Must compute mosaic number for this family.
3. Must show when mosaic number is realized, crossing number is not.

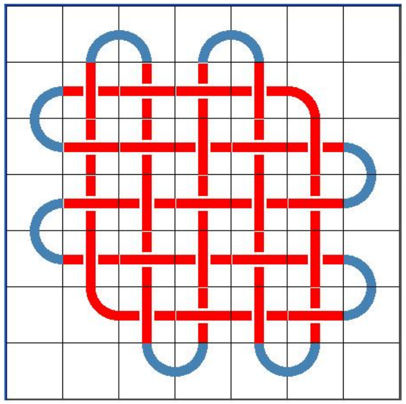
Act 1: Crossing Number



L_7

Act 1: Crossing Number

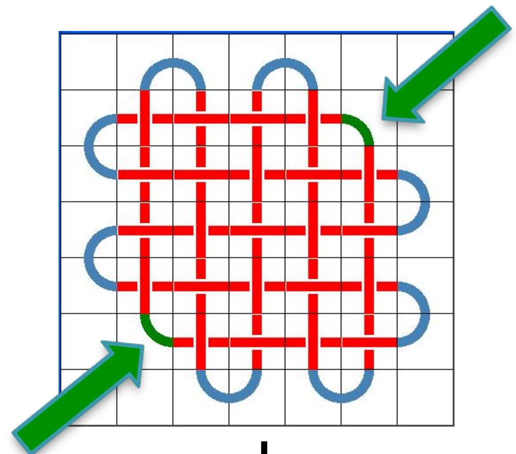
- $(2n\text{-}1)^2$ *inner* tiles



L_7

Act 1: Crossing Number

- $(2n-1)^2$ *inner* tiles
- $(2n-1)^2-2$ *crossing* tiles



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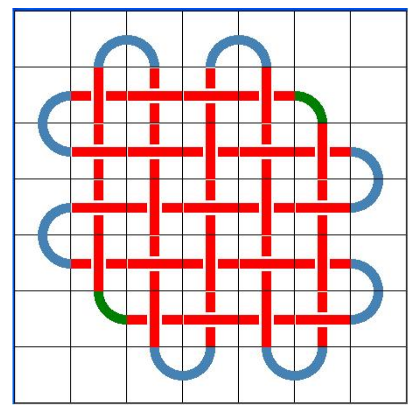
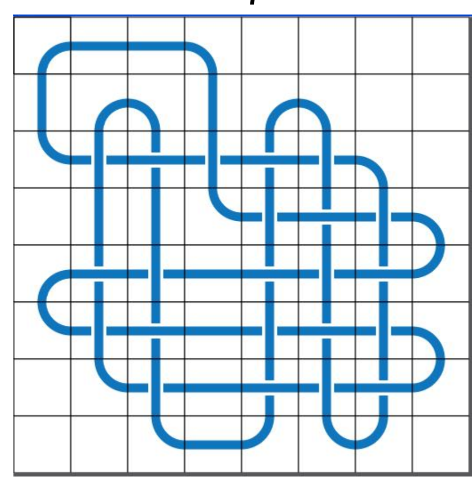


L_7

- *Make reduced alternating, remove one crossing*



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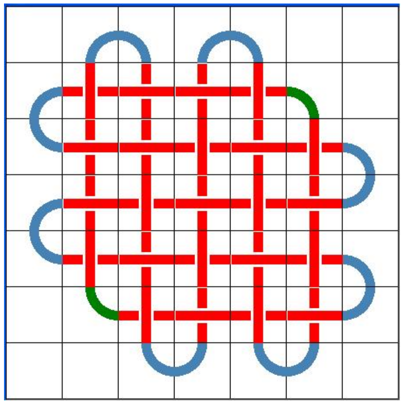
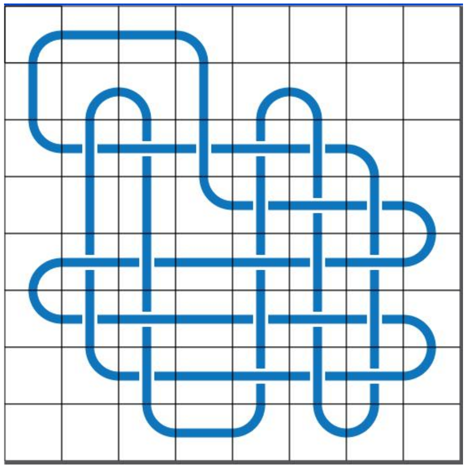


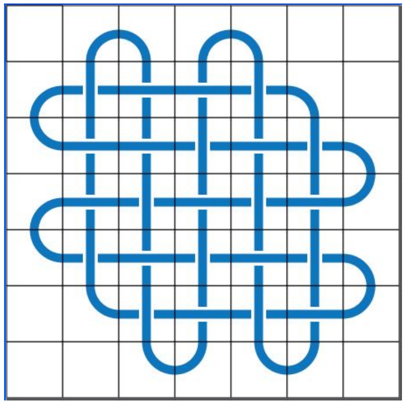
L_7

- *Make reduced alternating, remove one crossing*
- $c(L_{2n+1})=(2n-1)^2-3$

$(c(L_7)=22)$



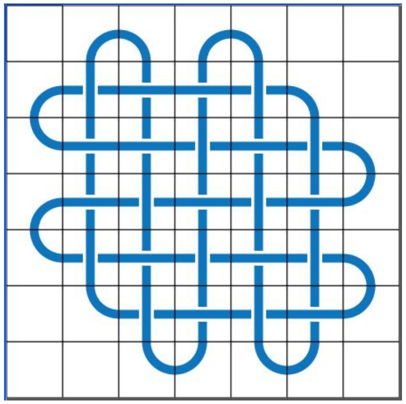
Act 2: Mosaic Number



L_7

Act 2: Mosaic Number

- Claim: $m(L_{2n+1})=2n+1$



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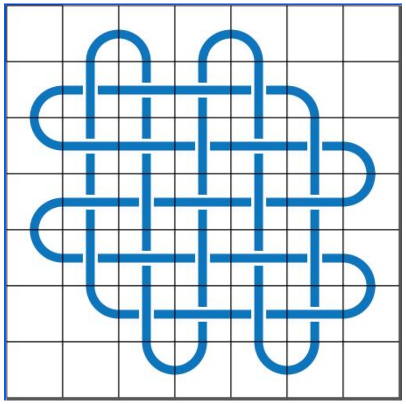

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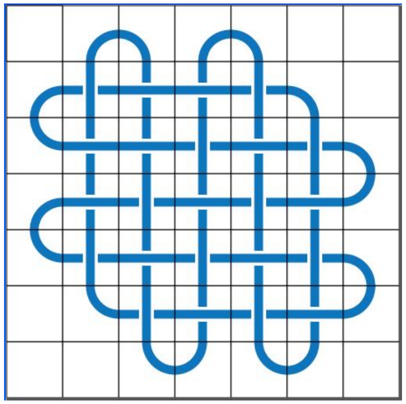
possible crossings



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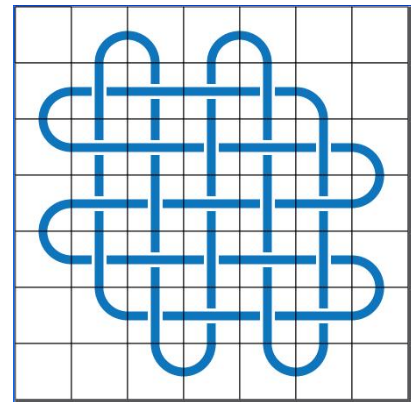

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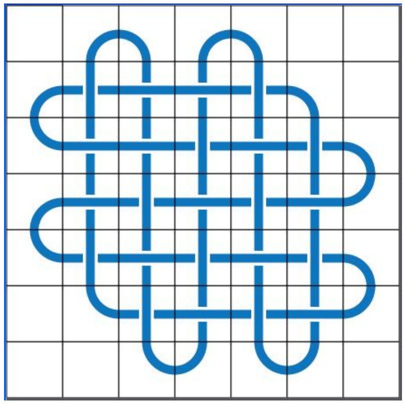

L_7

Act 3

- We must show that when the mosaic number is realized, the crossing number is not.

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- Important fact: L_{2n+1} is a reduced, alternating knot

Why are reduced, alternating Knots a big deal?

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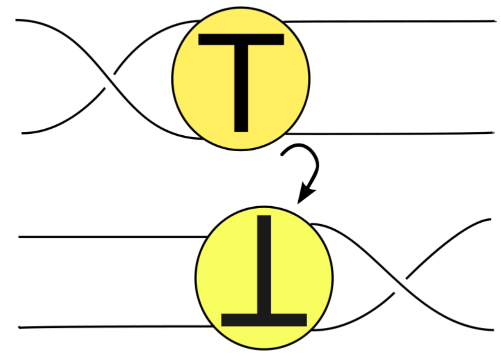
Tait Flyping Conjecture:

Given any two reduced alternating diagrams D_1 and D_2 of an oriented, prime alternating knot, D_1 may be transformed to D_2 by a sequence of *flypes*.

(Thistlethwaite & Menasco 1991)



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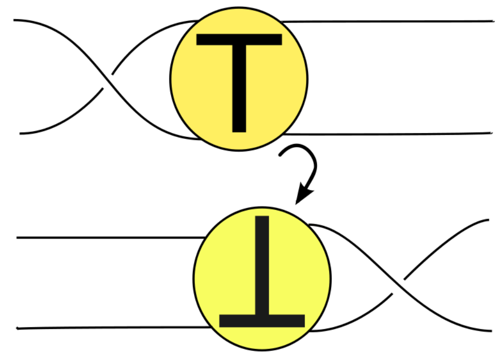
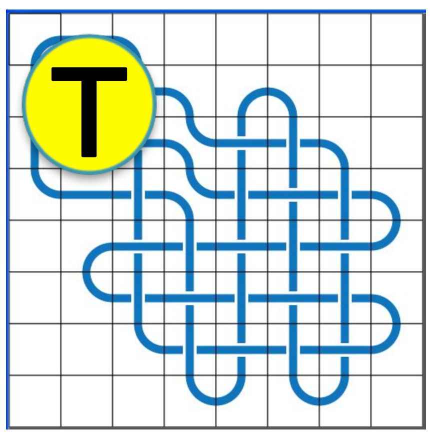
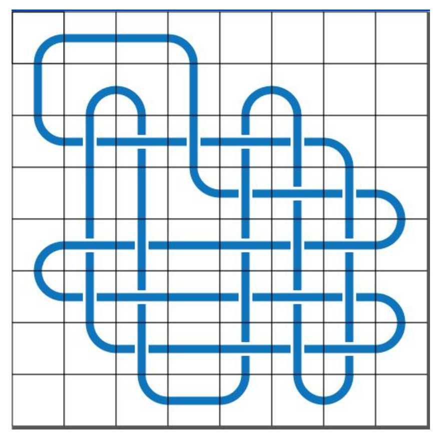


- L_7 has 1 possible flype





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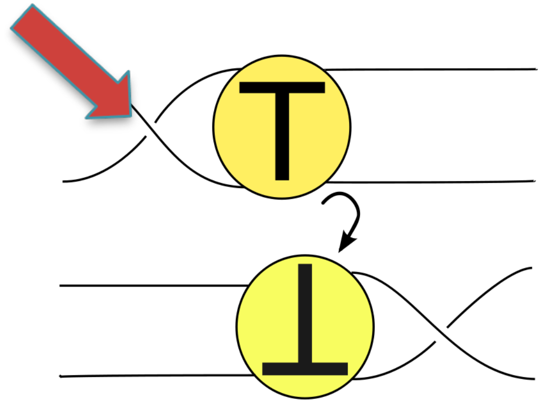

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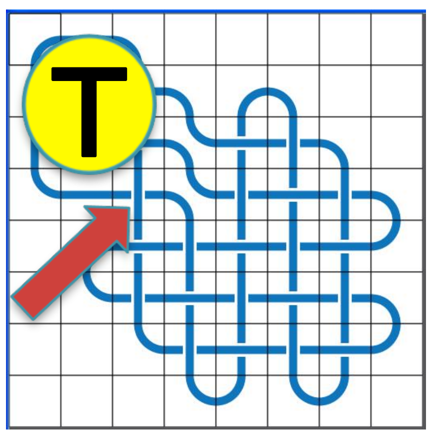

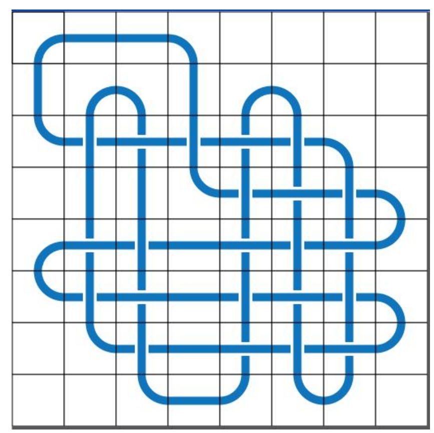

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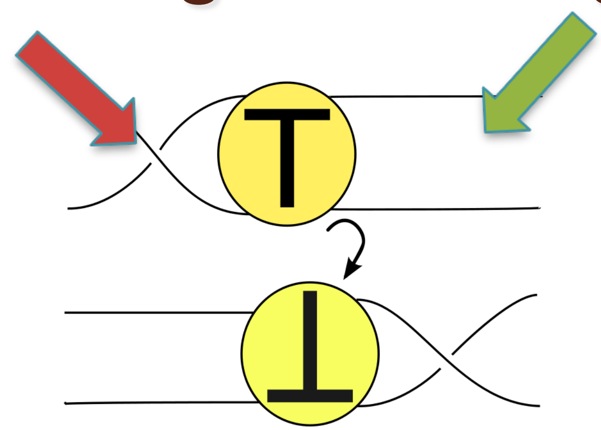

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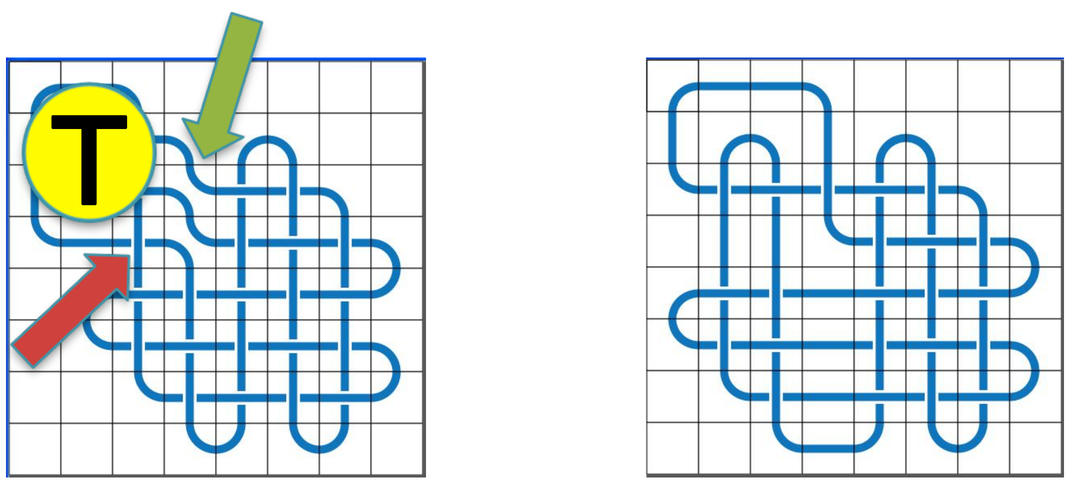

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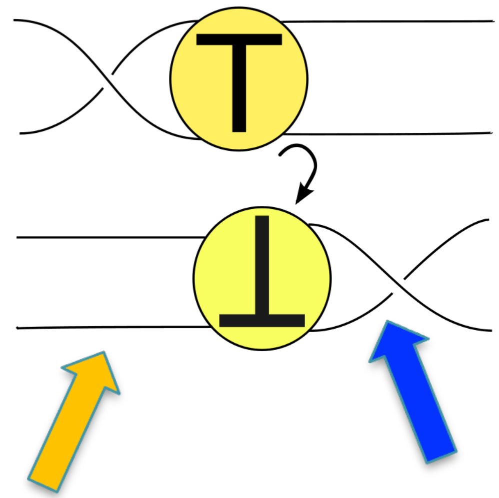

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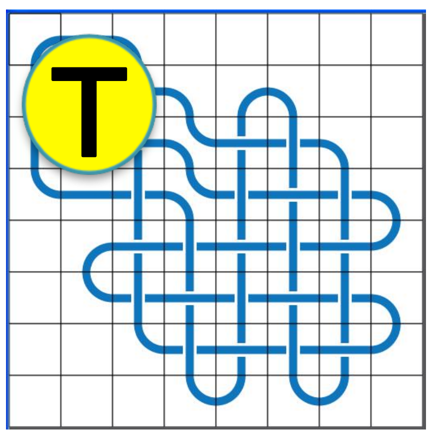

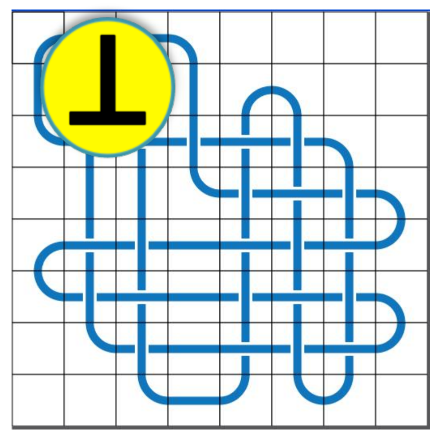

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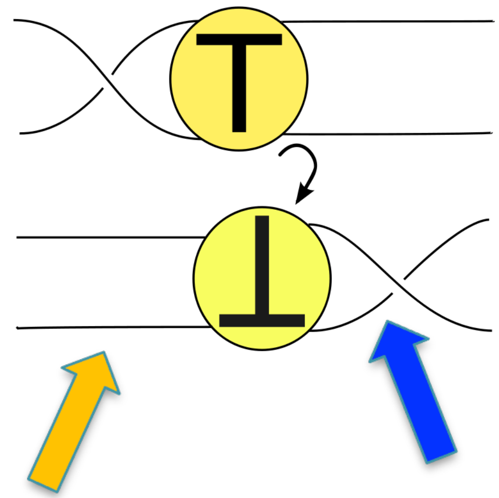

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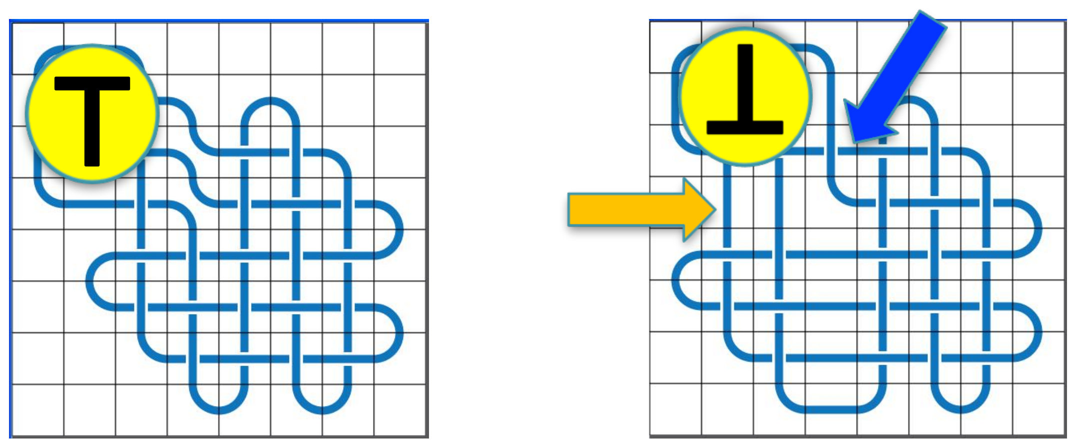

Conclusion…

- There are only two "versions" of L_{2n+1}

Conclusion…

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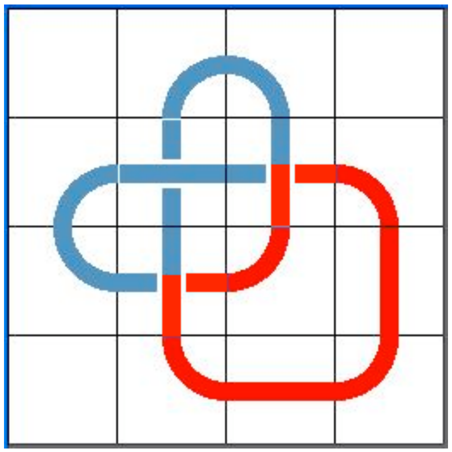
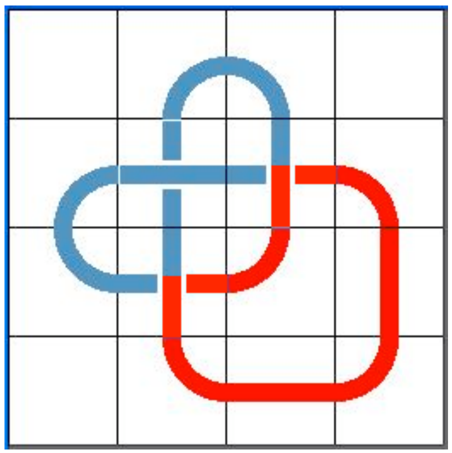
- How can the individual versions be placed on a mosaic board and maintain their crossings?

m-gons are preserved on sphere



Trefoil

m-gons are preserved on sphere



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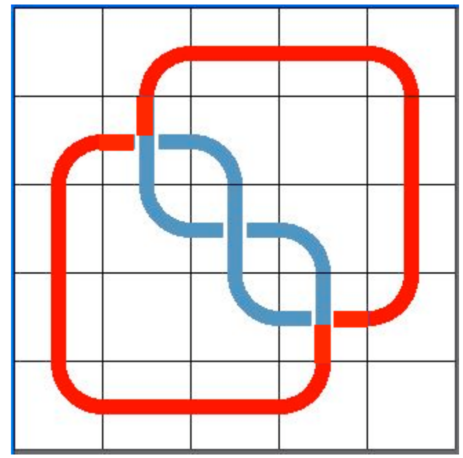

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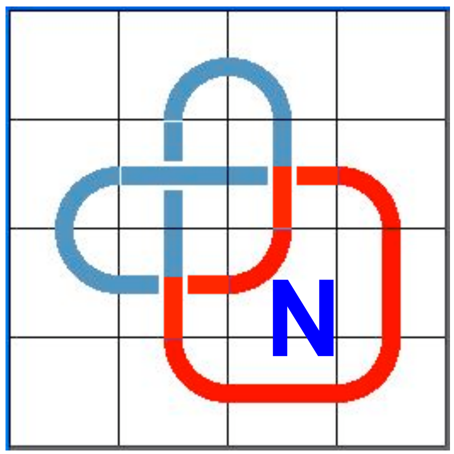
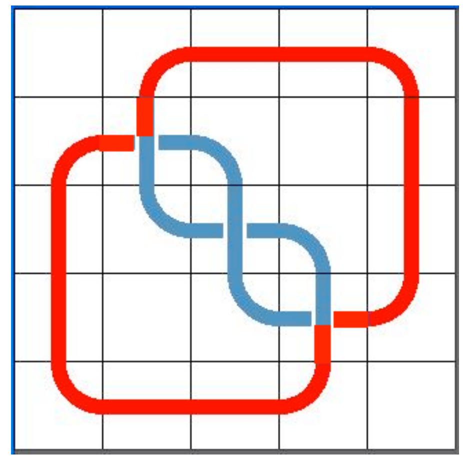
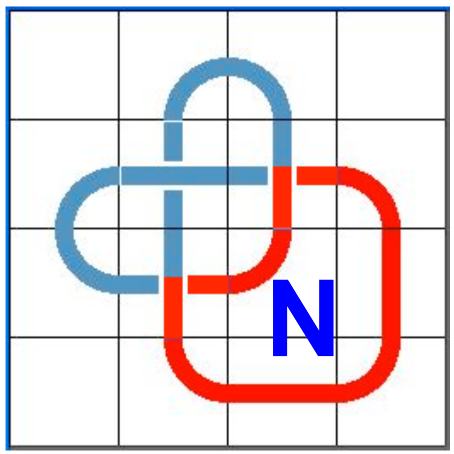


Trefoil



Trefoil

m-gons are preserved on sphere



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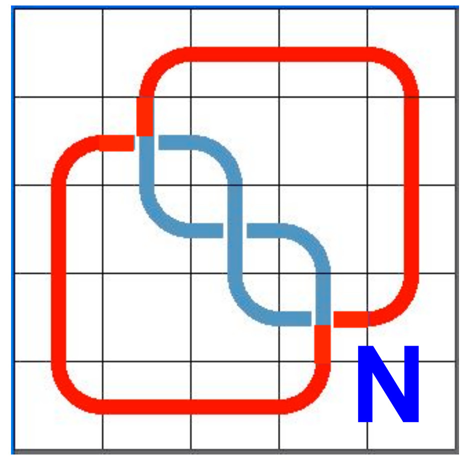
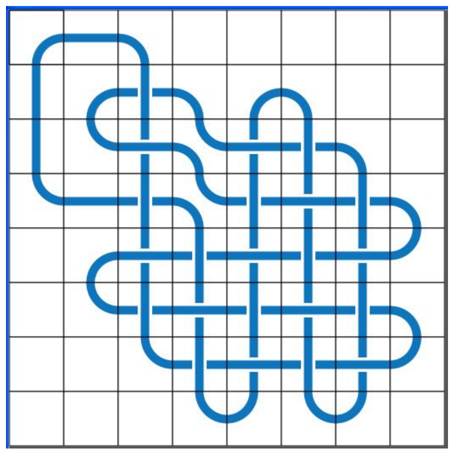


Trefoil

What about *m*-gons on L_{2n+1}?

n=3



<u>Lucky break 1:</u>
Only 2-, 3-, 4-, 5-, and (8n-11)-gons

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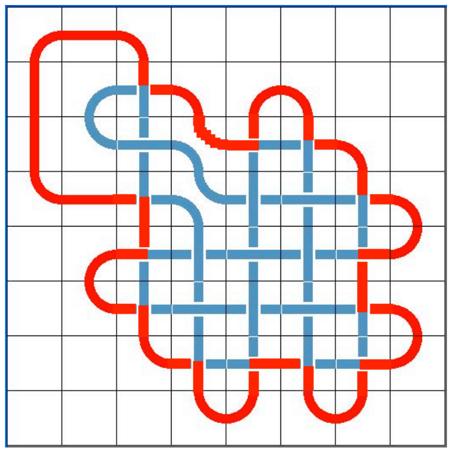

Lucky break 1:
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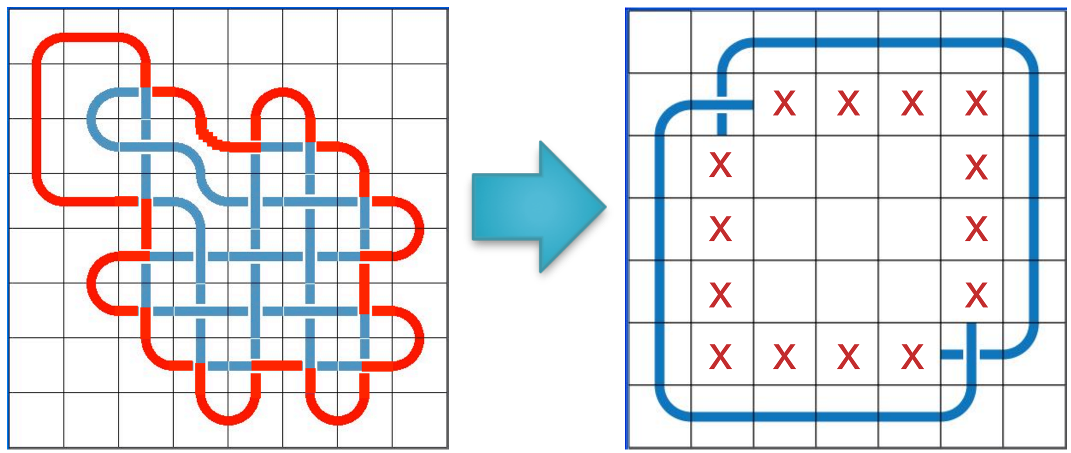
Ex: 2-gon won't work



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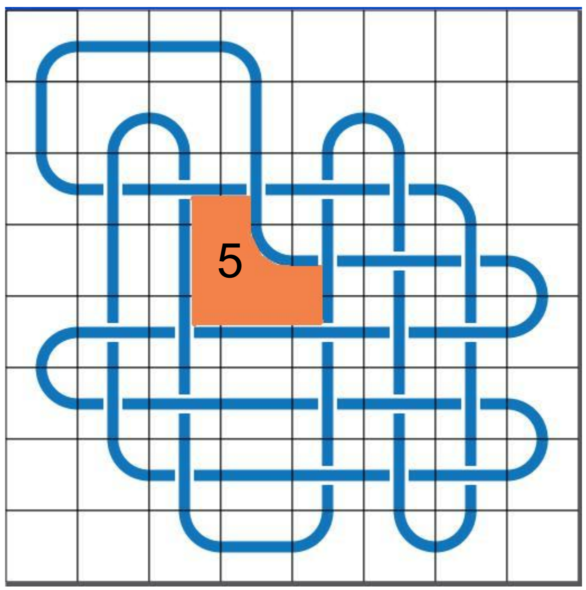
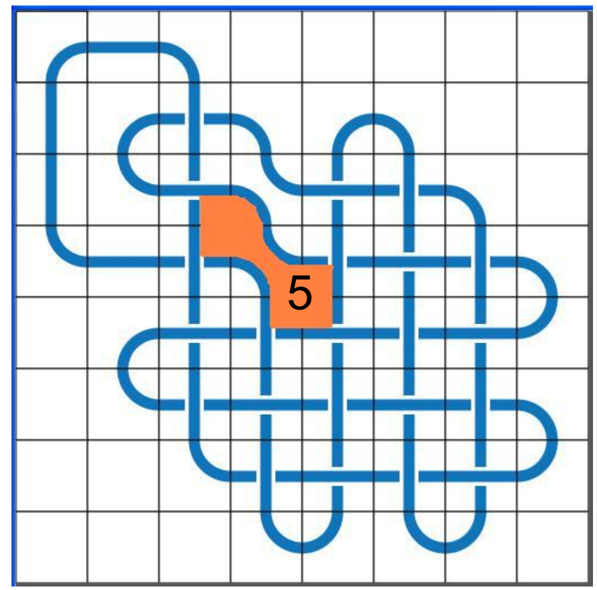


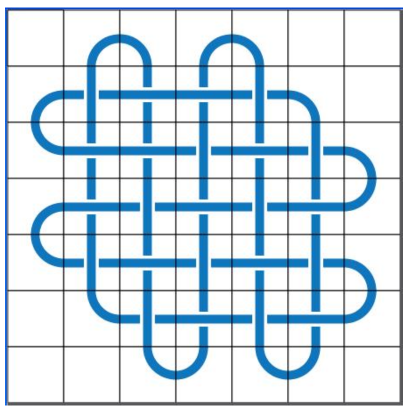


Lucky break 2:

Both reduced alternating projections have a **5-gon**.

Back to Act 3

- We know a non-reduced, non-alternating L_7 can fit on a 7x7 board, what about a reduced, alternating version of L_7?



L_7

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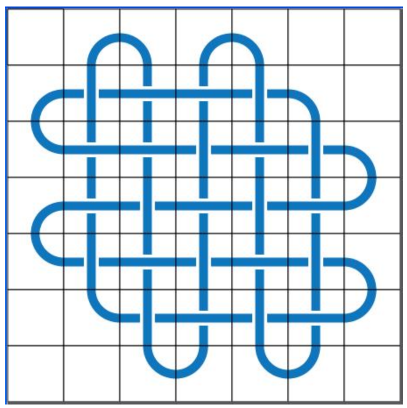
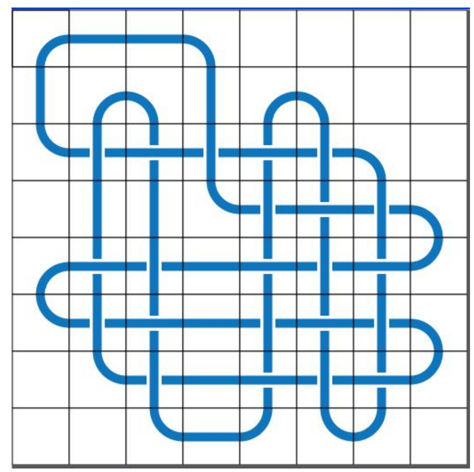


L_7



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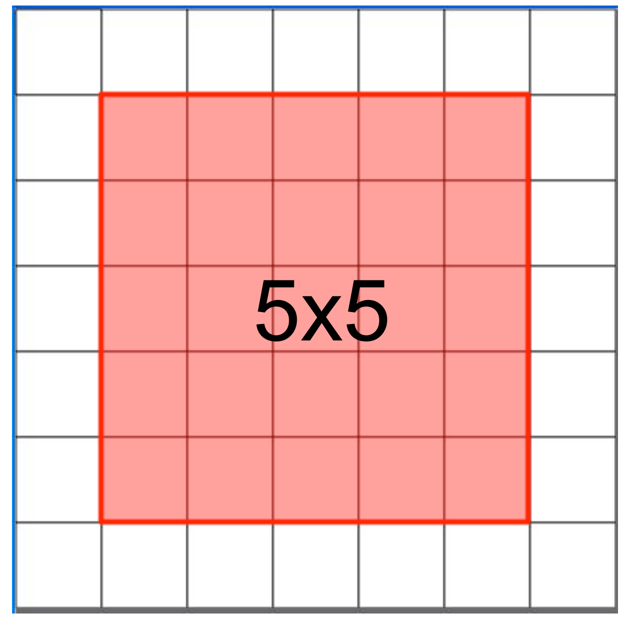

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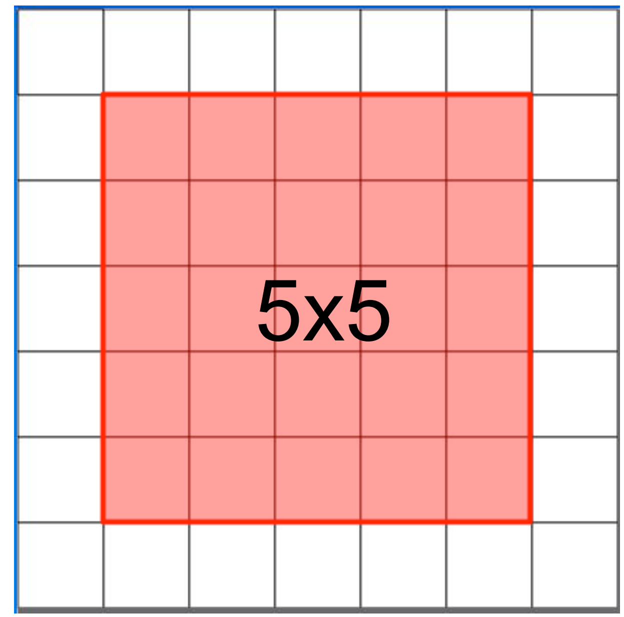

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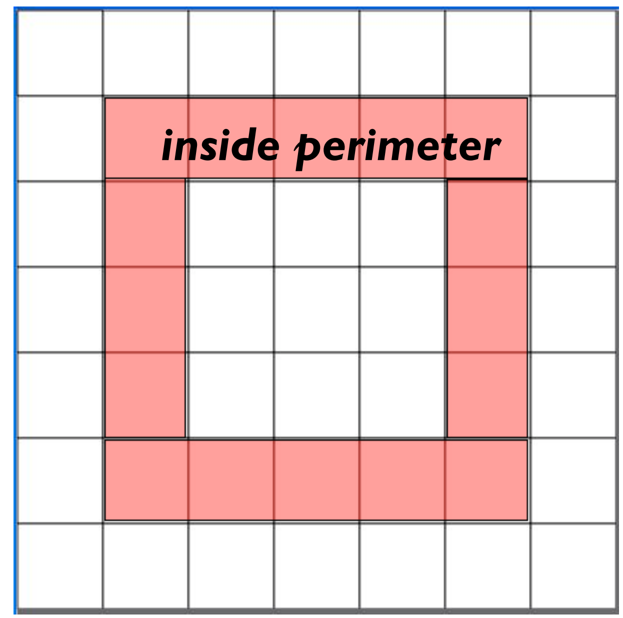

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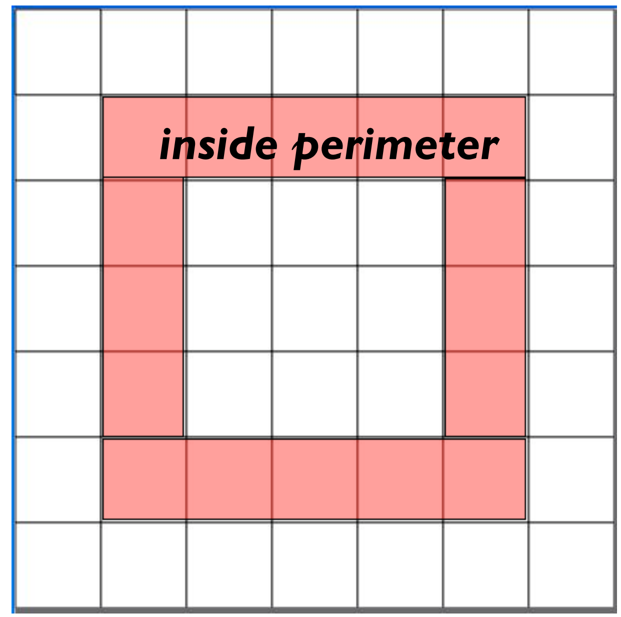

Where do the 3 non-crossing tiles go?

How can we place the three non-crossing tiles?

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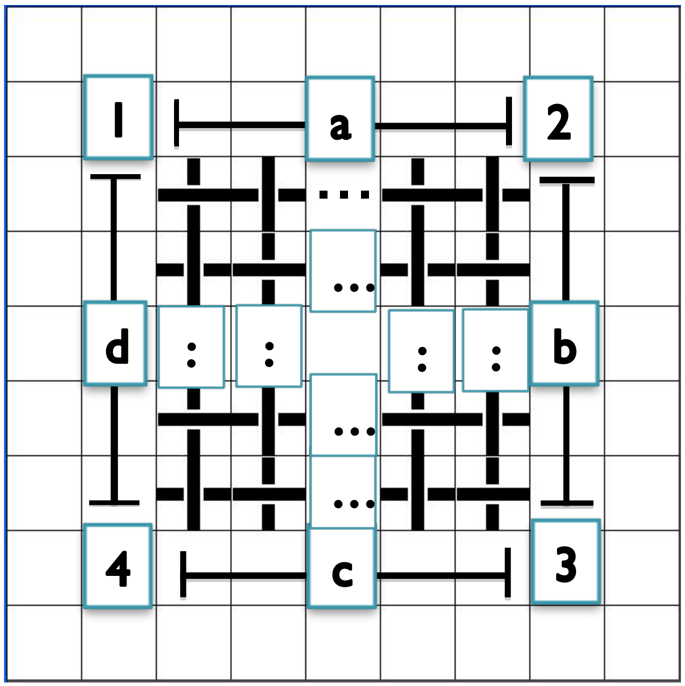

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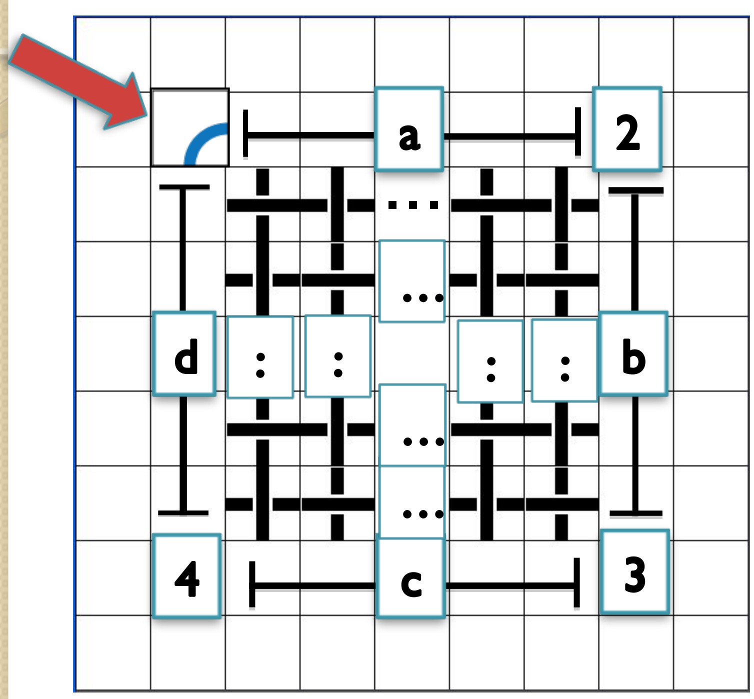

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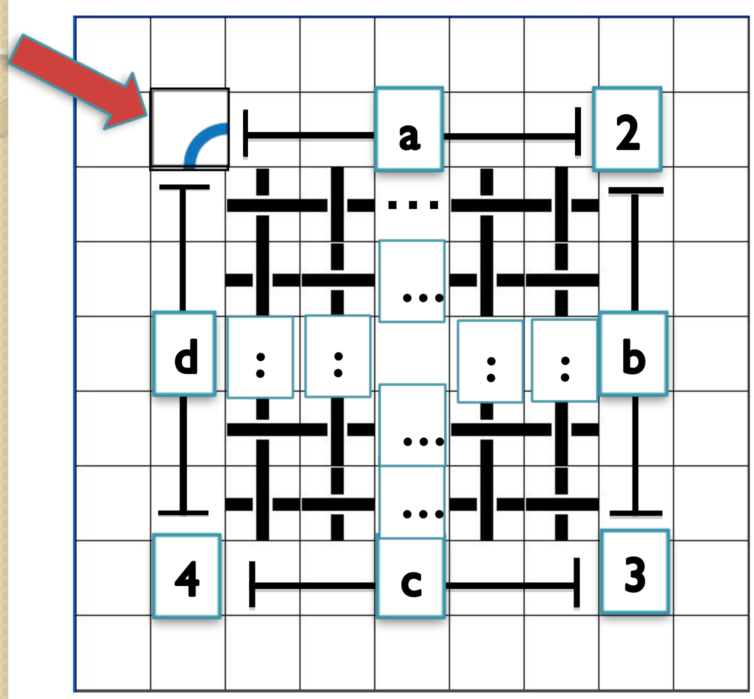

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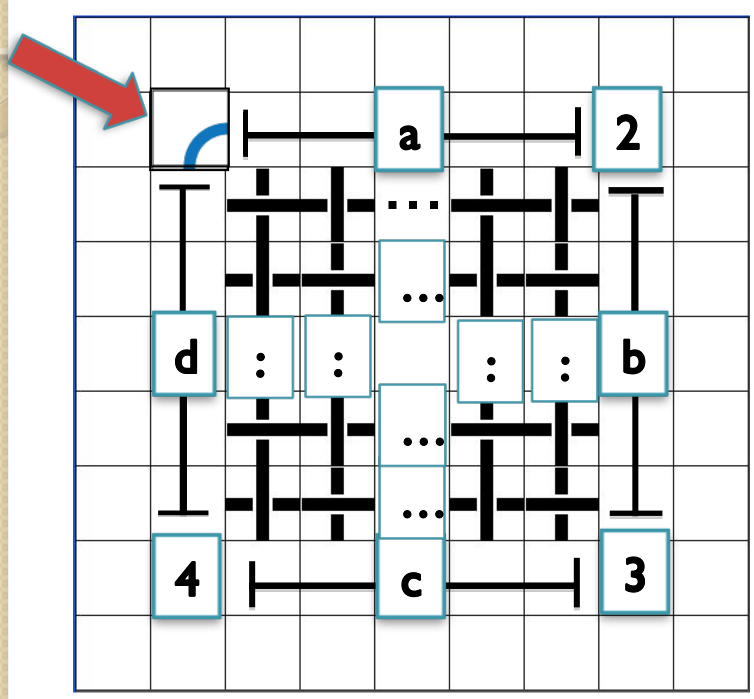

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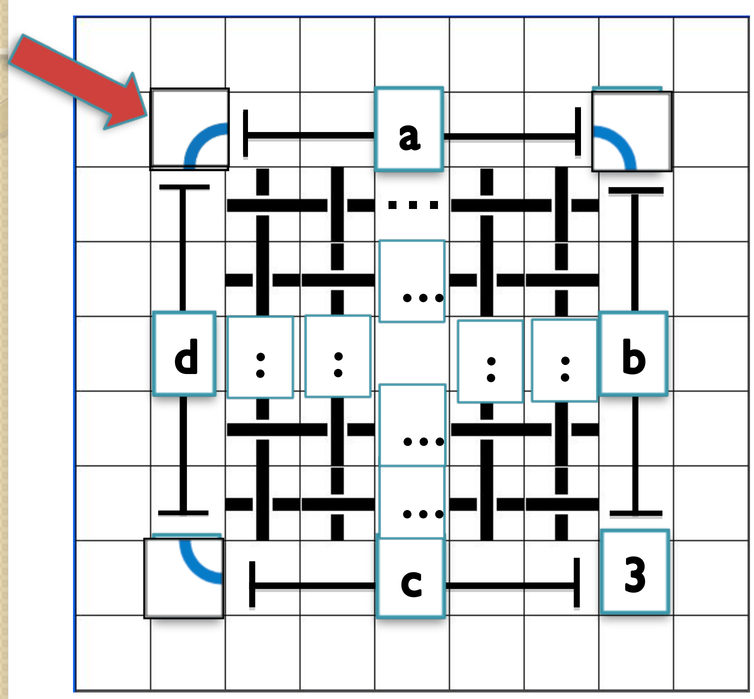

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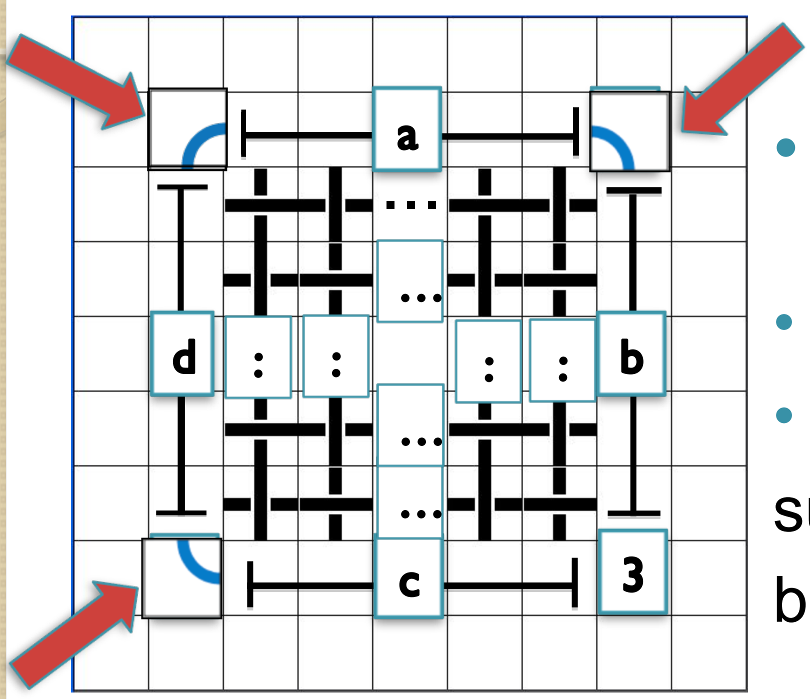

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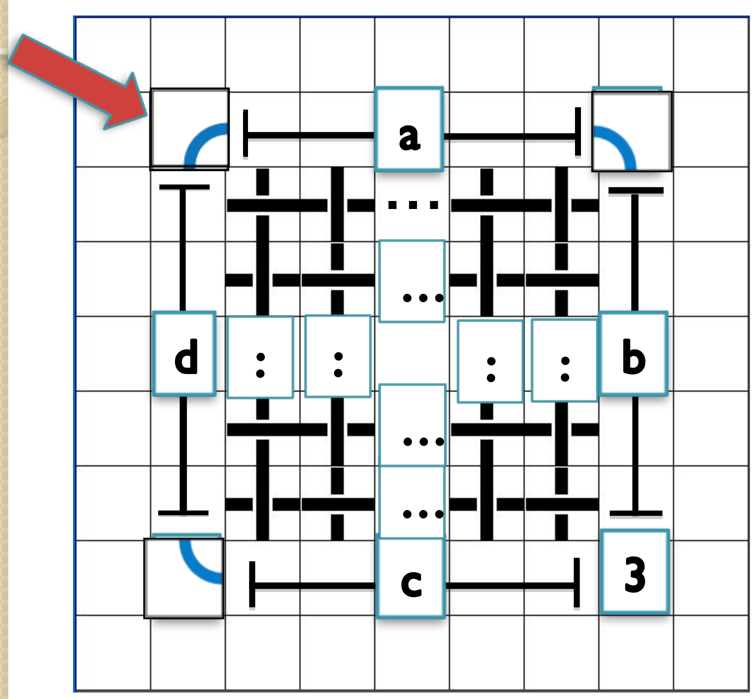


- ${}_7C_2$=21 ways to place other two non-crossing tiles
- Breaks into 6 cases
- Ex: both on a corner, suitably connected, no 5-gon
- Other 5 cases are similar, either no 5-gon or not suitably connected

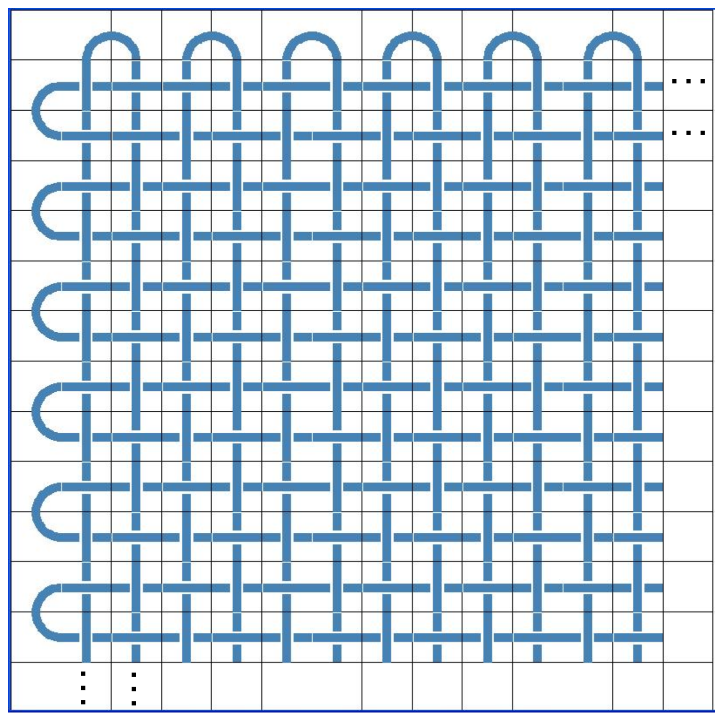
The close of Act 3…

- No matter how we tried, we could not get a reduced alternating L_{2n+1} to fit on a (2n+1)-mosaic board.

The close of Act 3…

- We found an infinite family of knots whose mosaic number is only realized when the crossing number is not.

L_{2n+1}



Why not even mosaic boards?

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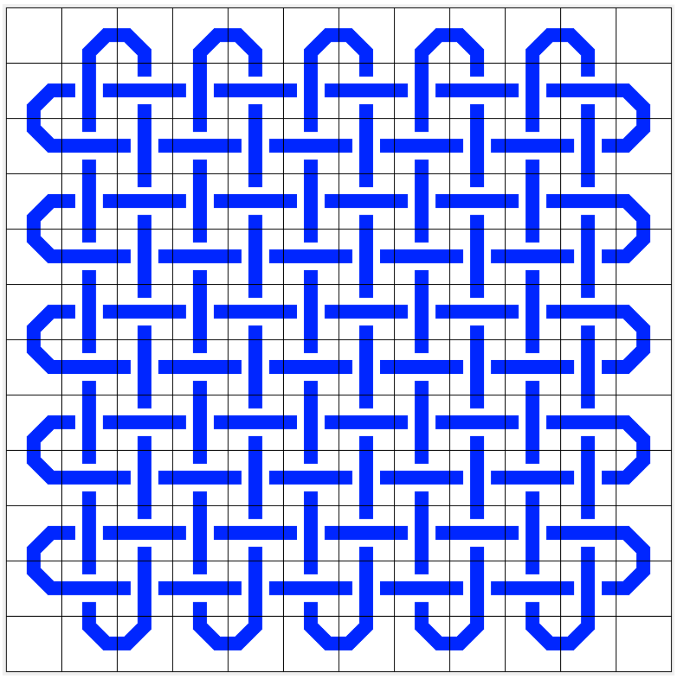
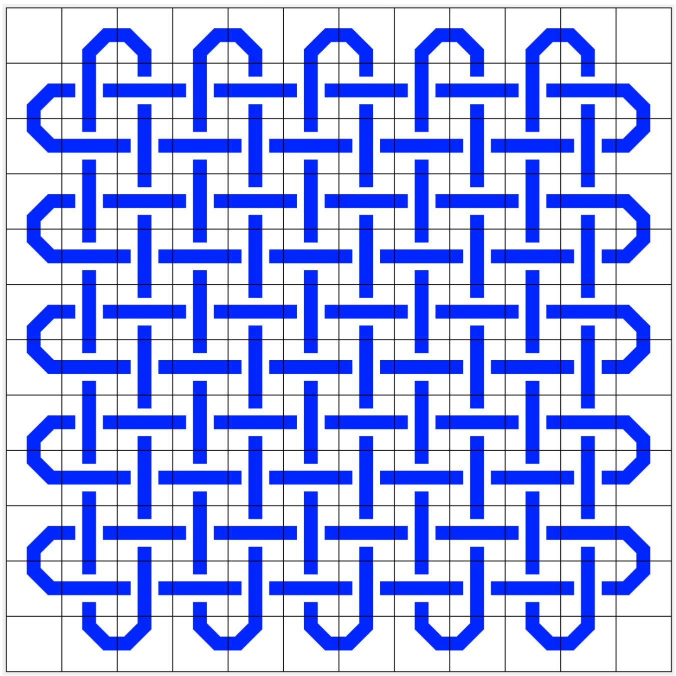
- L_{12}



Why not even mosaic boards?

- L_{12} – 10 component link



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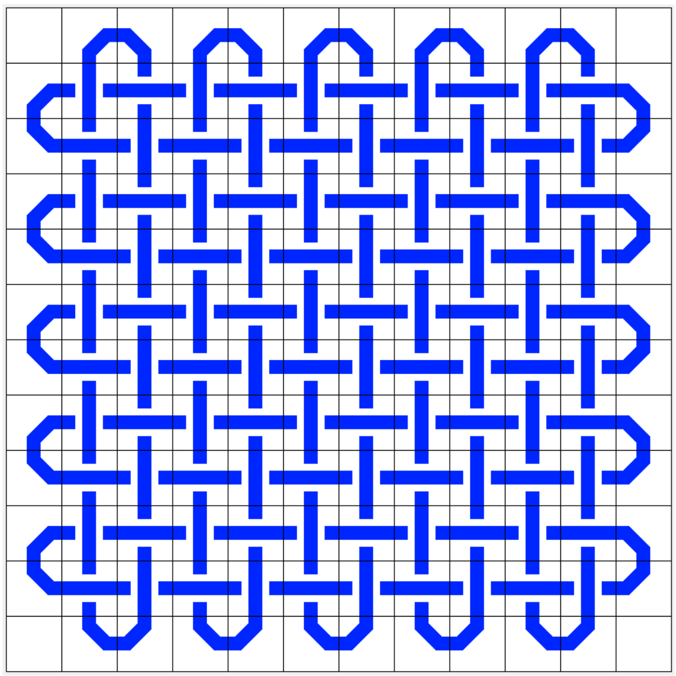

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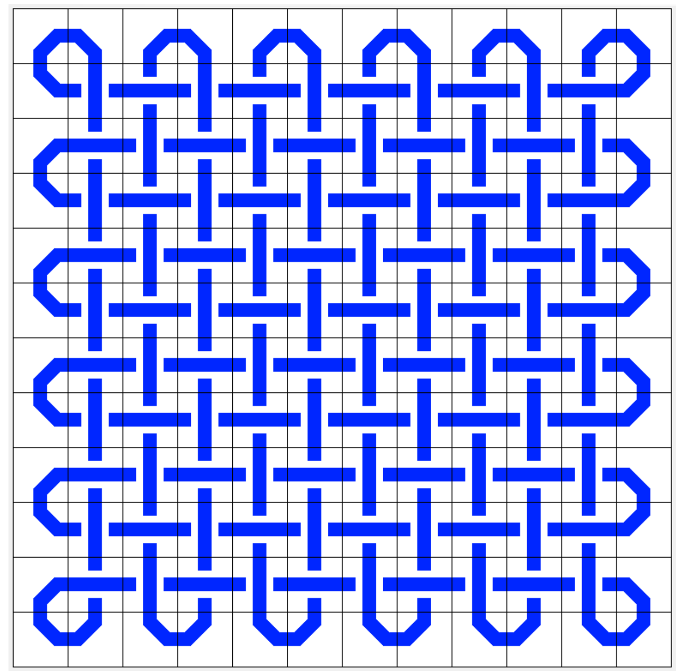

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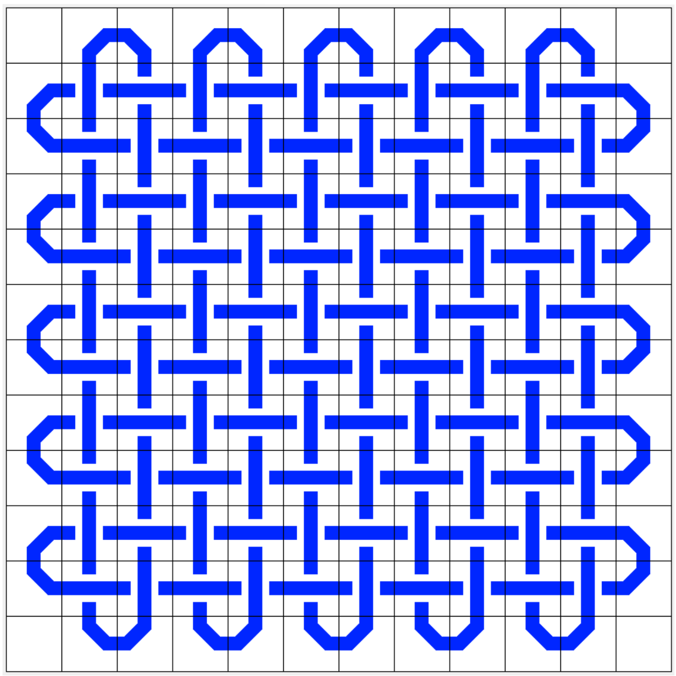
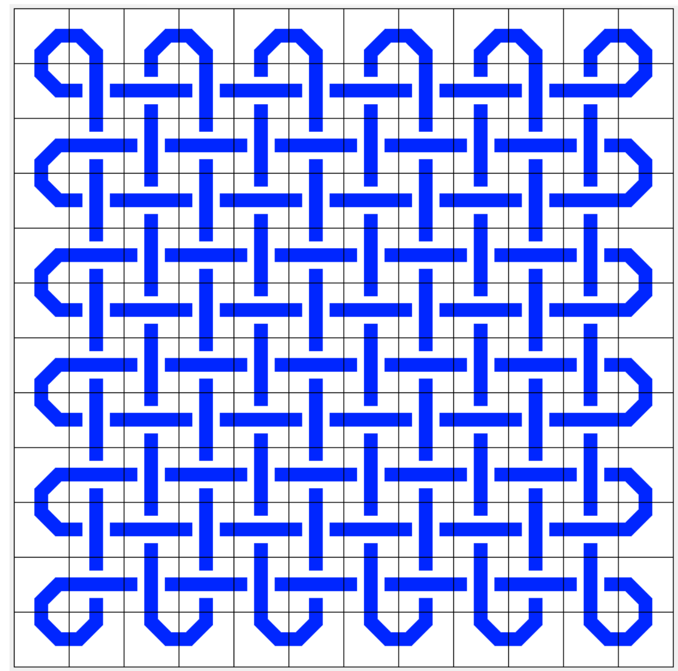
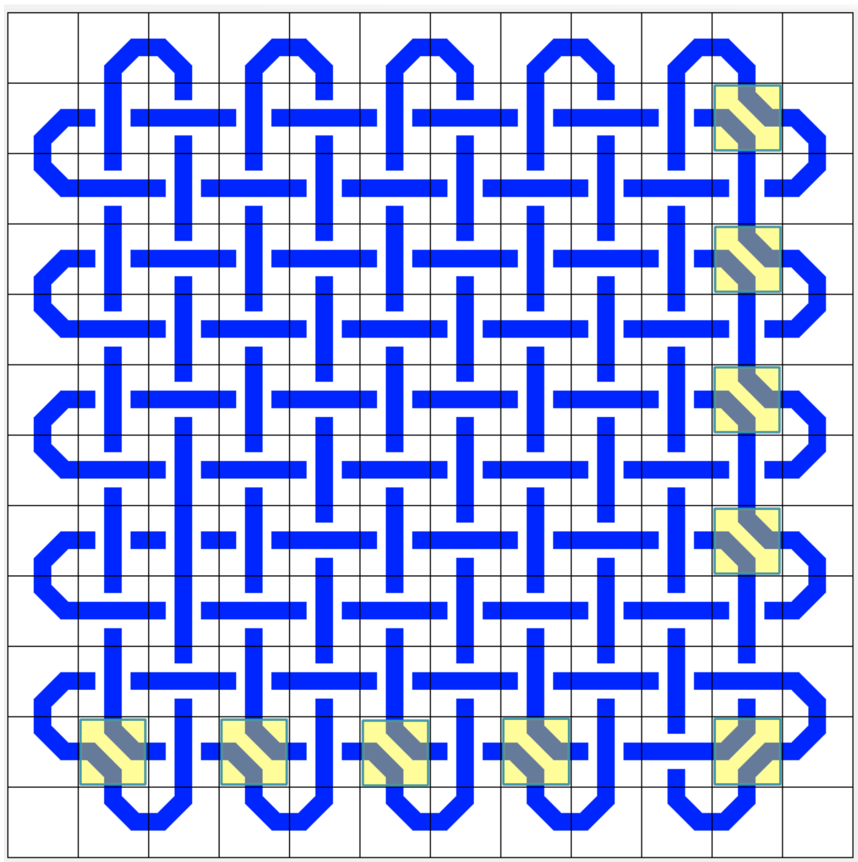


- L_{12} – 9 component link



New family of knots (L. & H.J. Lee)



n=6

- H_{2n}- helix knots

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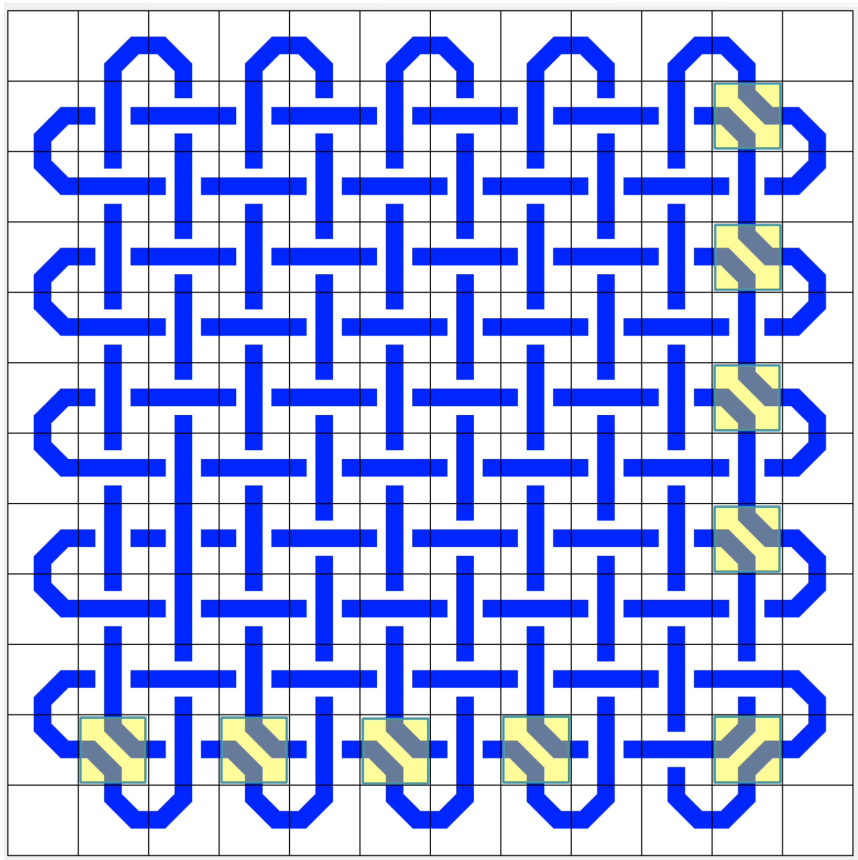
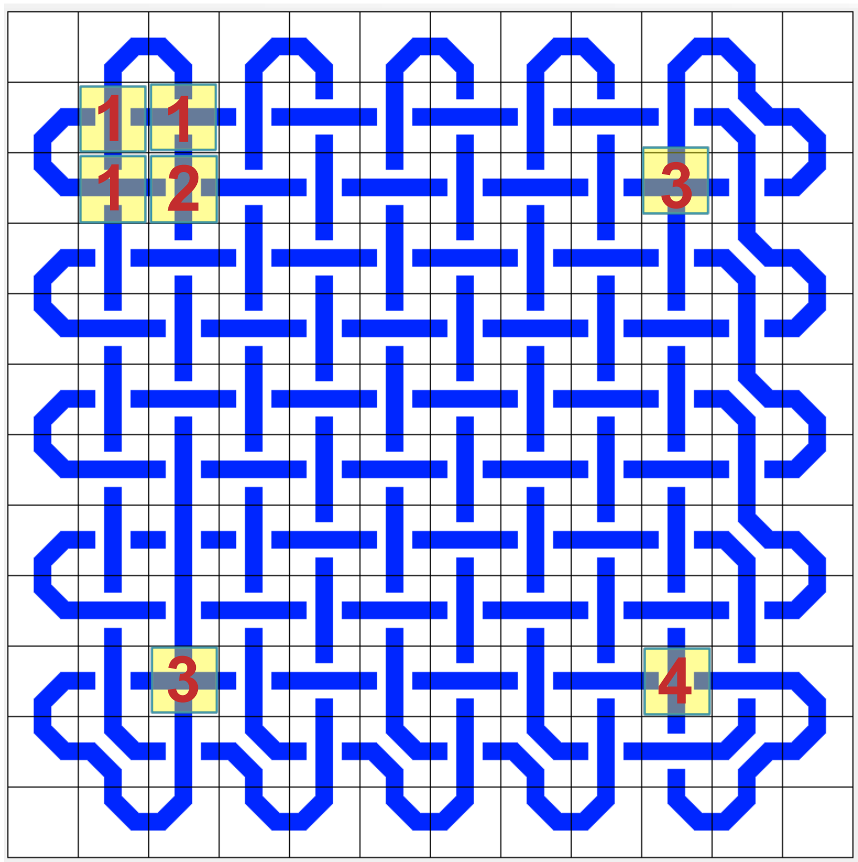


n=6

- H_{2n}- helix knots
  - Alternating knot
  - $m(H_{2n})=2n$
  - $c(H_{2n})=4n^2-10n+7$

Several families of knots (L. & H.J. Lee)



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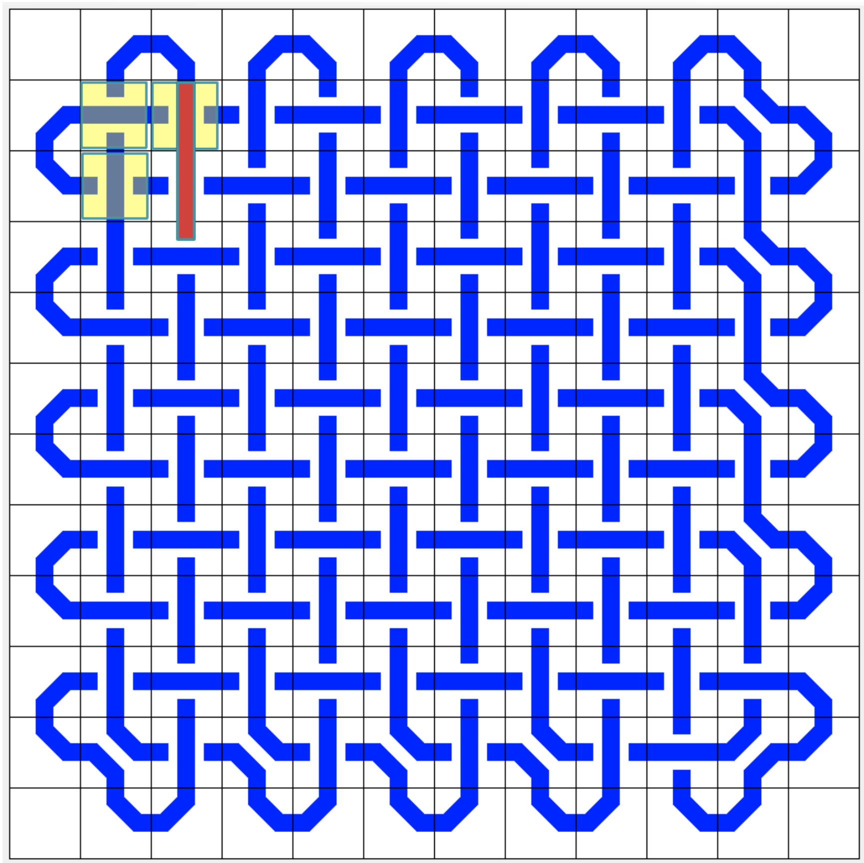
- H_{2n}- helix knots

Family of knot from H_{2n} (L. & H.J. Lee)

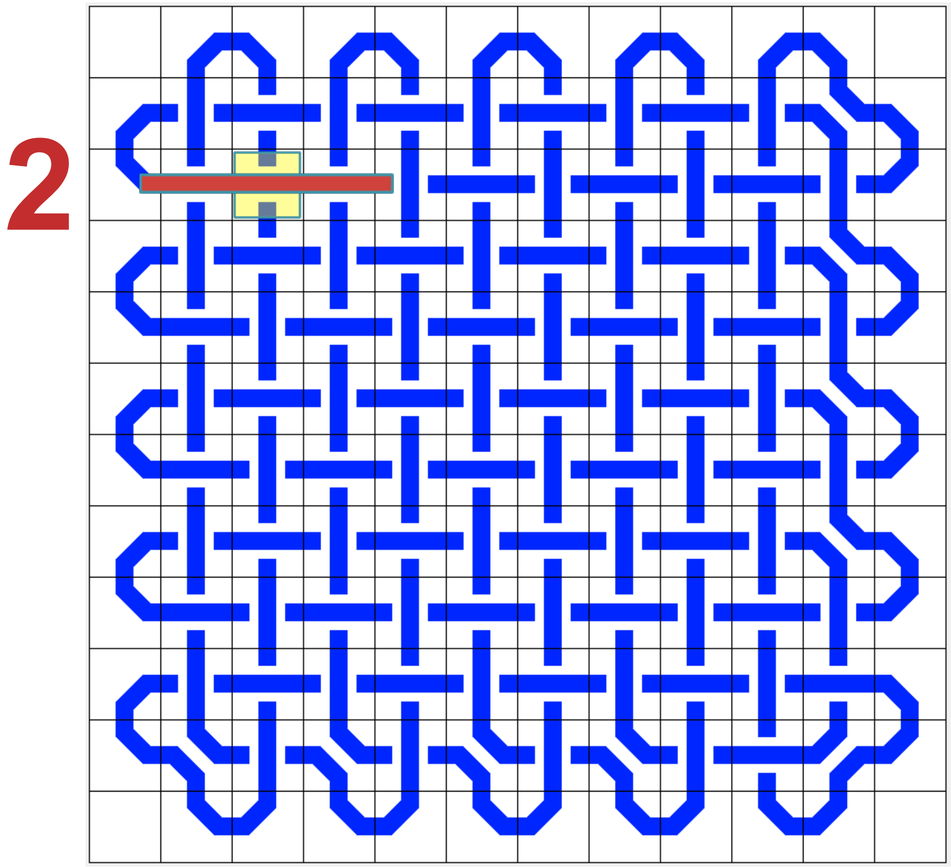
1



n=6

- HL_{2n}
  - Alternating knot, but non-alternating diagram
  - *$m(HL_{2n})=2n$*
  - *$c(HL_{2n})=4n^2-10n+6$*
  - *Mosaic number realized, crossing number not*

Family of knot from H_{2n} (L. & H.J. Lee)



n=6

- Haa_{2n}
  - Almost alternating knot
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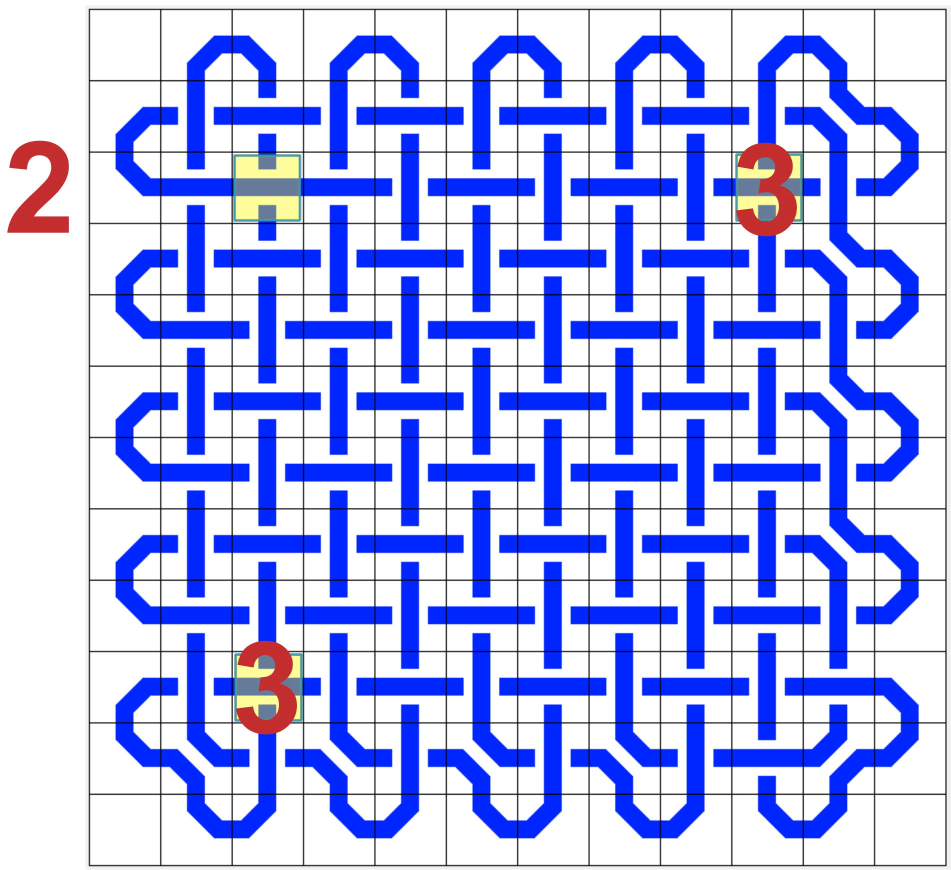
Family of knot from H_{2n} (L. & H.J. Lee)



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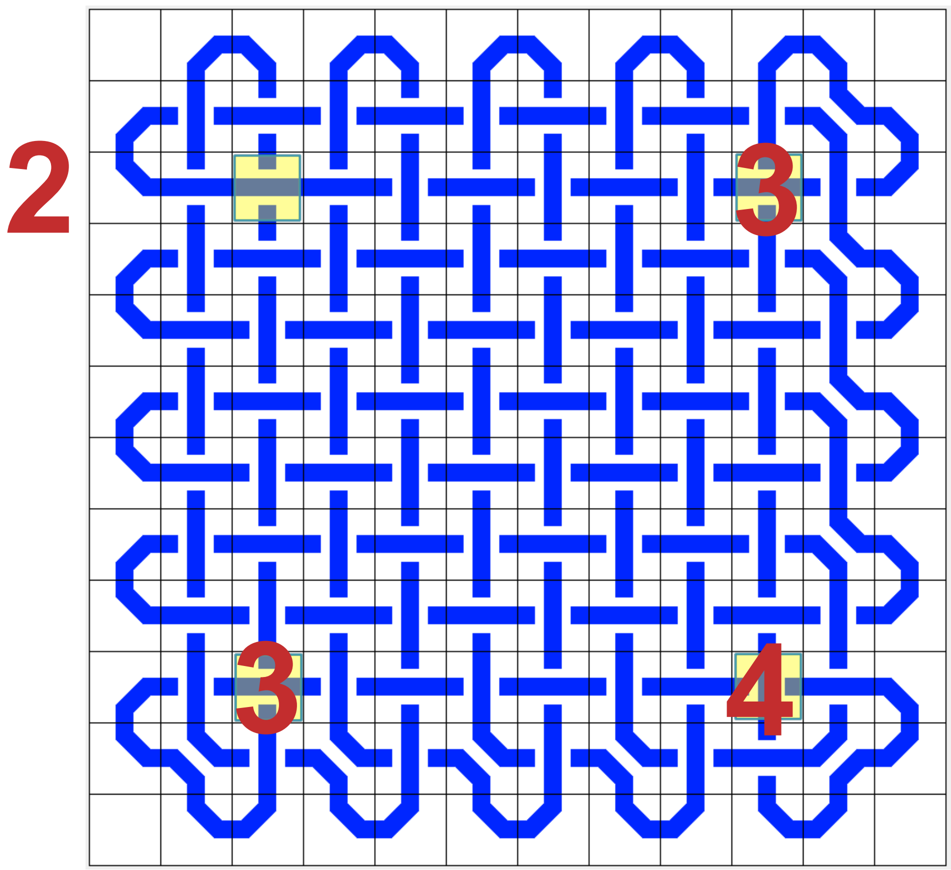

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- What is the mosaic number for (2,q)-torus knots?
- (p,p+1)-torus knots?
- Can the crossing number be used for determining the mosaic number?
- Does there exist a knot whose mosaic number is n, but whose crossing number is only realized on a mosaic board of size $n+2$?

Acknowledgements

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References

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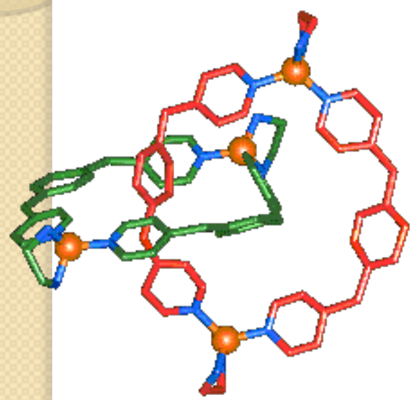
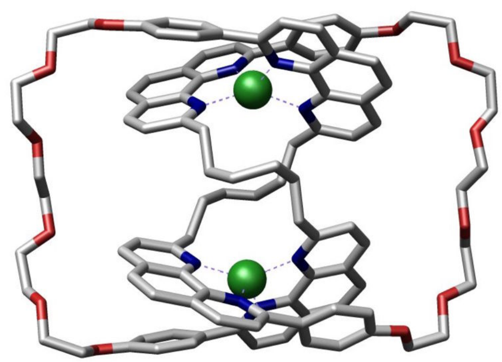
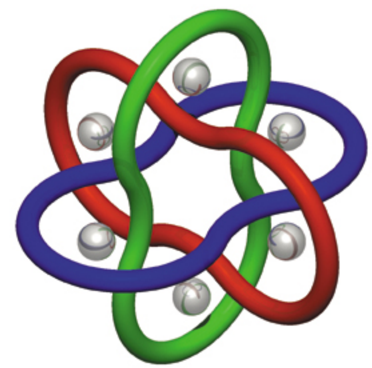
Why care about knots/links?



1961 Frisch & Wasserman



1993 Sauvage (80)



2004 Chichak et al.

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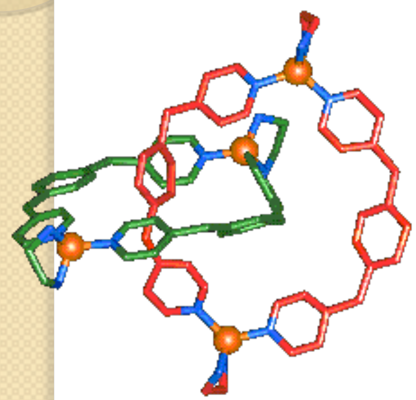

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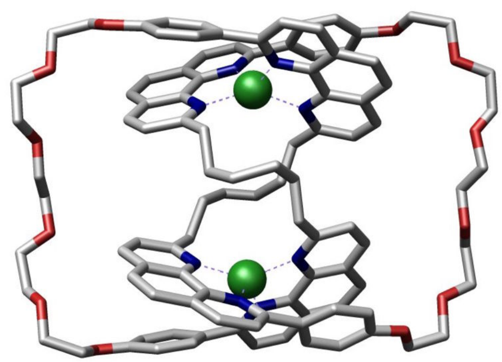

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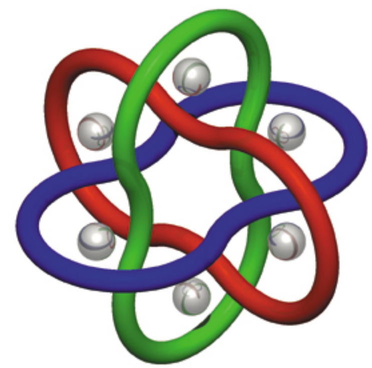

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2007 Fenlon – polyethylene trefoil (63)

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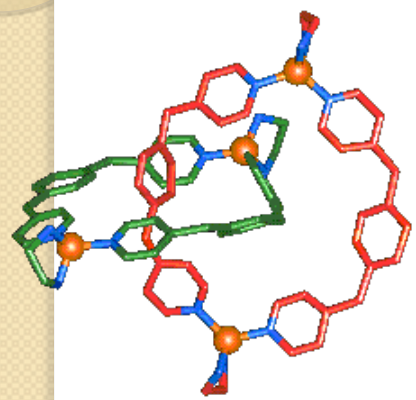

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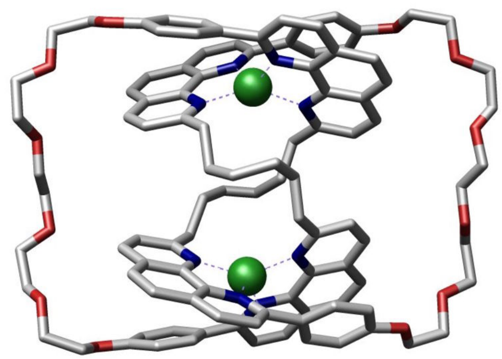

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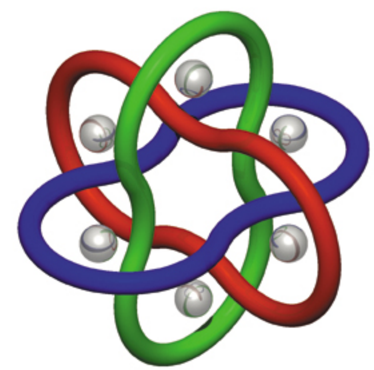

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UNIVERSITY OF LIVERPOOL

2010 nano-knots, two nanometers – around 30,000 times smaller than human hair