Omnipresence and Mathematical Reality

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1. Introduction

How much do we know about mathematical entities? According to an influential objection to platonism: surprisingly little, since we lack the causal access to them required for substantial knowledge.[1] Suppose, however, that this epistemic pessimism is mistaken and that we can know our accepted mathematical theories are true. If so, when we express theories like arithmetic and set theory, we succeed in correctly describing the specifically mathematical features of abstract entities like sets and numbers. On the resulting view, our mathematical knowledge is substantial but conspicuously limited, extending no further than the content of our mathematical theories. We can know that there are infinitely many numbers, that sets stand in certain membership relations, and that the universe of sets satisfies various axioms. But what else, if anything, can we know about abstract mathematical entities? For instance, do we know what *non*-mathematical relations they stand in (e.g., being intrinsic duplicates of one another) or what kinds of *non*-mathematical properties they instantiate (e.g., having qualitative properties)?

In *Parts of Classes*, David Lewis argues that we know little else about mathematical entities. Indeed, he claims that we are ignorant of some of the most basic facts about their non-mathematical nature. Remarking upon the question of where sets (or, alternatively, classes) are located relative to their members, Lewis (1991: 33) says:

> I don't say the classes are in space and time. I don't say they aren't. I say we're in the sad fix that we haven't a clue whether they are or whether they aren't. We go much too fast from not knowing whether they are to thinking we know they are not, just as the conjurer's dupes go too fast from not seeing the stooge's head to thinking they see that that the stooge is headless.[2]

In this chapter we take Lewis's epistemic worries about abstract mathematical entities seriously as we evaluate answers to a decidedly *non-mathematical* question about numbers and sets: where are they?[3] Among those who accept the existence of mathematical entities, the orthodox view is that of *transcendent platonism*, according to which such entities lack locations in space and time. But such a view is more usually presupposed than defended. It is therefore worth investigating why transcendent platonism should be preferred to *immanent platonism*, according to which mathematical entities are located. Moreover, what are the virtues and vices of *pervasive platonism*, which holds that mathematical entities are everywhere?

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[1] On epistemic objections to platonism, see Benacerraf (1973), Cheyne (2001), Field (1989), and Liggins (2010).

[2] Throughout, we treat sets and classes interchangeably. On Lewis' views of the relevant distinction, see Lewis (1991: 4).

[3] Bricker (2006) discusses the epistemic challenges surrounding our knowledge of the non-mathematical nature of sets.

Investigating the merits of pervasive platonism requires assessing a specific thesis of omnipresence: that mathematical entities are everywhere. [4] And, since philosophical debates about omnipresence have focused their attention on the omnipresence of divine beings, rather little consideration has been given to the mathematical omnipresence posited by the pervasive platonist.[5] In exploring this issue, we begin with a relatively schematic conception of both omnipresence and mathematical reality. We take *mathematical reality* to be the entirety of the iterative hierarchy of pure sets as described by ZFC.[6] (For the moment, we leave aside the case of impure sets such as {Socrates}.[7]) This conception has the significant virtue of supplying sufficient resources for any broadly naturalist philosophy of mathematics. It also affords us a way of talking about more familiar mathematical entities like numbers even though they turn out to be set-theoretic constructions.

For the moment, we assume a conception of omnipresence on which omnipresent entities are wholly located at each and every region of spacetime.[8] We largely set aside complications about classical worlds of space and time and throughout focus on spatiotemporal location.[9] We further suppose the truth of substantivalism about spacetime. This pattern of location should be distinguished from the view on which entities are exactly located at the maximal sum of spacetime and only partially located at sub-regions. Roughly speaking, this conception of the omnipresence of mathematical reality is analogous to views regarding Aristotelian universals as wholly located in each of their instances.[10] Later, we will consider whether an alternative conception of the omnipresence of mathematical entities might be a better option for platonists.

Our central question of where mathematical entities are located also points toward a broader one about whether other abstract entities like properties and propositions are located in space and time. One way to pursue this topic is to insist that whatever holds true of some abstract entities must hold true for others. Granted that view, we could reasonably infer that properties and propositions are without locations if mathematical entities are and *vice versa*.[11] While there are many versions of platonism, some of which seek to uphold this uniform picture

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[4] Pervasive platonism should be distinguished from what Carmichael (2016) calls deep platonism, which is holds abundant properties to be located where their bearers are located.

[5] On omnipresence theses regarding non-divine entities, see Cowling and Cray (2017)

[6] We leave open whether there is a mereological sum of mathematical entities or whether mathematical reality can only be designated using plural quantification.

[7] Pure sets have no non-sets in their transitive closure.

[8] This conception of omnipresence requires what Inman (2017) calls ubiquitous entension, following the apparatus set out in Hudson (2005) and discussed in Hudson (2009).

[9] For a discussion of the complications regarding abstract entities once we distinguish temporal and spatial location, see Hoffman and Rosenkrantz (2003) and Juvshik (2020).

[10] On the location of universals, see Armstrong (1989: 99)

[11] If a metaphysician adopts the right suite of views, certain theses of omnipresence regarding abstracta will follow. For instance, if we one defends the view that properties are immanent universals and also upholds the view that some physical property like, say, having *mass-energy* is instantiated by all spatiotemporal regions, such a property would be omnipresent. We take such a view to tell us little about how we should view the nature of mathematical reality and its relation to spacetime.

of abstract reality, the assumption of abstract homogeneity is difficult to defend and our confidence in principles expounding on the one true nature of abstract reality should be limited. In this domain, reasonable controversy and subtle complications abound. There is ontological disagreement over whether certain entities like properties, propositions, and numbers exist. And there is further disagreement about which, if any, of these entities are properly counted as abstract entities as well as what it would mean for them to be so categorized. We take our best bet for a plausible verdict on the location of mathematical entities to be gleaned by focusing on them in theoretical isolation from other putatively abstract entities.

A final qualification: in what follows, we assume questions regarding the location of numbers are meaningful and that proposed answers are no less meaningful. Rylean suspicions about the coherence of asking where numbers are located should not be dismissed lightly and, in a paper evaluating the view on which they are everywhere, these suspicions are liable to grow with each page. A Rylean response to Lewis's epistemic worries about mathematical entities might also seem tempting partly for its brutal elegance: we don't know non-mathematical things about mathematical entities because there is nothing non-mathematical to know about such things. The following discussion, which explores but does not endorse a view of numbers as omnipresent, is perhaps the best kind of argument against the accusation of a category mistake. If you find yourself convinced of the falsity of pervasive platonism, we can at least be assured it is meaningful.

2. The Case for Transcendent Platonism

Why should we believe the transcendent platonist thesis that mathematical entities are without spatiotemporal location? As Lewis argues, any view about the location of such entities cannot be inferred from the content of our best mathematical theories. Although claims about the location of sets are—when the topic of their location is not being ignored by mathematicians—treated as "unofficial axioms," they enjoy no significant support from mathematical practice. The adoption of transcendent platonism is therefore a matter of habit rather than principle:

> Set theory has its unofficial axioms, traditional remarks about the nature of classes. They are never argued, but are passed along heedlessly from one author to another. One of these unofficial axioms says that the classes are nowhere: they are outside of space and time. But why do we think that? (Lewis 1991: 31)

When philosophers take up the question directly, the first argument one likely encounters is tied to the category of *abstract entity* with mathematical entities cited as a paradigm example. According to what we can call the *definitional argument*, sets, numbers, and any other mathematical entities are without location because they are abstract rather than concrete entities, and what it means for an entity to be abstract is simply for it to be the kind of thing that lacks spatiotemporal location. As Lowe (1998: 513) notes, the lack of spatiotemporal location is one of several ways that philosophers have proposed to analyze the term *abstract*:

> On the first conception [of abstract entity], the term 'abstract' is used in opposition to the term 'concrete', with concrete entities being thought of as existing in space and time (or

> at least in time), while abstract entities are correspondingly thought of as being nonspatiotemporal in nature.[12]

Although lacking spatiotemporal location is regularly claimed as a feature of various abstract entities, analyses according to which abstractness is nothing more and nothing less than lacking location are implausible. Those who hold fictional characters or musical works to come into existence at certain times can, without evincing incoherence or confusion, assert them to be abstract. Conversely, physical posits that are non-spatiotemporal by virtue of being more fundamental than spacetime would not necessarily be identified as abstract. The same holds true for divine beings who are claimed to be "outside" of spacetime. Rather than chase down the issues raised by the definitional argument, we are content to set it aside.

When pressed to develop an argument for transcendent platonism that does not rest on conceptual fiat, a natural starting point is the premise that we never perceptually detect them in our spatiotemporal world. Perhaps this is reason enough to believe them to be without location.[13] But, even though seeing or touching something is good evidence that it is spatiotemporally located, our apparent inability to see or touch mathematical entities is shakier ground to infer that they lack locations. This *argument from imperceptibility* provides support for transcendent platonism only if we were antecedently justified in believing that mathematical entities are the sorts of things that, if were they located, we would be able to see or touch. Once we take seriously Lewis' epistemic pessimism about sets and other mathematical entities, it is unclear what supports this assumption regarding their counterfactual perceptibility. Although our belief in the existence of mathematical entities is justified on account of their possession of certain mathematical and structural features, none of these features bear upon the question of mathematical entities' intrinsic nature—e.g., whether they would resist acceleration, would reflect infrared radiation, or have the intrinsic disposition to be perceptible by humans. Commenting on this peculiar dimension of our ignorance regarding sets, Lewis (1991: 33) says:

> Another unofficial axiom says that classes have nothing much by way of intrinsic character… Are all singletons exact intrinsic duplicates? Or do they sometimes, or do they always, differ in their intrinsic character? If they do, do those differences in any way reflect differences between the character of their members? Do they involve any of the same qualities that distinguish individuals from one another? Again, we cannot argue the case one way or the other, and if we think we know that classes have no distinctive intrinsic character, probably that's like thing we know the stooge is headless.

The argument from imperceptibility is only as compelling as our knowledge of the intrinsic nature of mathematical entities is secure. But since our knowledge of mathematical entities' intrinsic natures is just as shaky as our knowledge about their location, the argument from

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[12] The sort of view he has in mind is defended in Grossman (1992: 7): "An abstract thing is a thing which is neither temporal nor spatial. A concrete thing, on the other hand, is a thing which is temporal and/or spatial. The "and/or" is necessary because of the possible that there are temporal things, like thoughts, which are not spatial."

[13] Cf. Maddy (1980).

imperceptibility cannot lend significant support to transcendent platonism. We are therefore obliged to concede that if numbers and sets were in spacetime, they might simply be the kinds of things that cannot be perceived.

Lewis' observations about our knowledge of sets and mathematical entities threaten to scuttle various other arguments for transcendent platonism that depend upon tendentious assumptions regarding non-mathematical features. For example, suppose that mathematical truths are necessary and, furthermore, that the world could have been without any spatiotemporal reality—e.g., a non-spatiotemporal realm of disembodied intellects. If we hold that mathematical entities are spatiotemporal and, indeed, essentially spatiotemporal, they could only be contingent beings. This would, however, violate the assumed necessity of mathematics and therefore motivate holding mathematical entities to be without location.

Notice, however, that this line of argument hangs upon the assumption that if mathematical entities were actually located, then they would be essentially located.[14] But what would justify the assumption that numbers can only exist when they have locations? Why couldn't numbers be the sorts of things that are spatiotemporally located in worlds of spacetime but without locations in other worlds? Once again, our argument for transcendent platonism is only as plausible as our claim to knowledge of the non-mathematical nature of mathematical entities—in this case, their essential features—but such knowledge is precisely what we are seeking to secure.

Perhaps the strongest argument for transcendent platonism issues from an allegedly implausible consequence of holding mathematical entities to be located: if mathematical reality has a location, then mathematical reality must therefore has a shape. But such a view seems absurd. And, unlike the arguments above, this argument does not seem to depend upon claims about the intrinsic nature of mathematical entities. Instead, it hinges upon general principles about location and shape.[15]

Entities with certain patterns of locations do seem to have shapes, and they do seem to have those shapes in virtue of the patterns of occupation they bear to spacetime. For instance, if mathematical reality occupied all and only a spherical region of spacetime, it would seem plausible to conclude mathematical reality has the property being spherical, and this would be a good reason to reject the view that mathematical reality exhibits such a pattern of occupation. But is the pervasive platonist required to endorse the thesis that, if an entity stands in any pattern of occupation, it must therefore instantiate some shape property? To assess this claim, we ought to first distinguish between, on the one hand, shape properties like *being a sphere* or *being T-shaped* and location properties like *occupying disconnected regions of spacetime* or *occupying three point-sized regions within the five meters of each other*. Obviously, objects with patterns of occupation instantiate location properties, but such properties need not be shape properties. We evince no conceptual confusion when we ask whether something instantiating the property of *occupying three point-sized regions within five meteres of each other* has a shape. Similarly, it seems an open question whether an omnipresent entity that is wholly located at every region of

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[14] On mathematical contingency, see Rosen (2002).

[15] We set aside a different strategy for blocking this argument: insist that shape properties are intrinsic and therefore appeal to Lewis's skepticism. On extrinsicality and shape, see Skow (2007).

spacetime has a shape. We should therefore be careful to mark that denying entities have shape properties is different from denying they instantiate location properties.

A commitment to pervasive platonism entails that mathematical reality has a determinate and remarkable location property: being wholly located at each and every region. But the pervasive platonist can consistently deny that such a location property entails any consequences regarding the shape of mathematical reality. Although the fact that mathematical reality is wholly located at both spherical and cubical regions (among arbitrarily many others) might invite the conclusion that it has incompatible shape properties, the pervasive platonist can claim that the location properties of omnipresent entities are perfectly coherent precisely because entities that are wholly located at arbitrarily many regions have no shape properties. To be sure, such a view raises questions about the nature of shape properties and their dependence upon both location and modal properties, but it is far from clear that pervasive platonists are especially worse off for it. (We will, however, consider an alternative response to such arguments in Section 5 that sidesteps such worries altogether.)

3. Theoretical Virtues and Transcendent Platonism

If transcendent platonism cannot be deduced from some more basic premises about abstract or mathematical entities, perhaps it can be supported by general theoretical considerations like parsimony. This path for defending transcendent platonism hinges on showing the view to enjoy a greater share of theoretical virtues than any form of immanent platonism. Such a strategy has its limitations: transcendent platonists typically assert the non-spatiotemporal nature of mathematical entities as something close to obvious, but if it is only vindicated through carefully weighing theoretical virtues and vices, transcendent platonism would instead turn out to be a highly tentative hypothesis.

What theoretical virtues might be used to vindicate transcendent platonism? Probably not parsimony. Transcendent and pervasive platonism share the same ontological commitments by requiring the existence of spacetime and mathematical reality. Both views also share an ideological commitment to concept of location even if transcendent platonist hold that only concrete entities are located.[16] With respect to their formulation, the views are on all fours: numbers are nowhere or they are everywhere. A difference in complexity might be traced to the fact that pervasive platonism requires a theory of location complex enough to assert mathematical reality is wholly located in multiple regions, but some transcendent platonists already require such complexity for other reasons and will also assert mathematical reality is wholly absent from the very same regions. On balance, it does not seem as though considerations of parsimony license a preference for transcendent over pervasive platonism.

As noted above, most platonists are transcendent platonists. So, if the theoretical virtue of conservativeness favors minimally revisionary views, this might be cited as a relevant advantage. Notice, however, that in those cases where conservatism is plausibly invoked, we tend to have some kind of epistemic footing in the relevant domain even if our account of it is in disrepair. The upshot of Lewis's arguments regarding the non-mathematical nature of mathematical entities is that we seem to have no credible footing at all in this domain. An

[16] On the varieties of parsimony, see Cowling (2013).

appeal to conservatism should therefore be deemed unsatisfactory as the basis for defending transcendent platonism.

Alongside virtues like simplicity and conservativeness, theoretical vices—specifically, arbitrariness—have been brought to bear on the question of where sets are located. Of singleton sets—sets with a unique member like {Socrates}—Lewis says the following:

> If Possum's singleton [i.e., {Possum}] were elsewhere than Possum himself, it would presumably be an obnoxiously arbitrary matter where it was. If it's in Footscray, why there instead of Burwood? But perhaps Possum's singleton is just where Possum is. Perhaps, indeed, every singleton is just where its member is.

Here, Lewis notes that certain theses about the location of sets are viciously arbitrary, but singles out as non-arbitrary (or at least importantly less arbitrary) the thesis according to which singletons are located where their sole members are located. This thesis is not a rival theory about the location of mathematical entities; it concerns exclusively singletons and remains silent on the case of impure sets with multiple members as well as pure sets. (Interestingly, this thesis does entail that at least some sets are omnipresent—e.g., the singleton of the sum of all spatiotemporal regions is located precisely everywhere.) It is tempting, however, to see whether this thesis might be expanded to provide a general formulation of immanent platonism that builds upon it as a non-arbitrary foundation. Unfortunately, doing so requires a series of burdensome choices in order to provide any guidance about the location of mathematical reality. Not only are we left with the open question of where impure sets with multiple members are located (not to mention pure sets), we now owe an account of sets built from singletons that occupy spacetime. For example, is {Socrates, Ø} wholly located where Socrates is or only partly located there?[17] So, even if the case of {Possum} can be addressed without lapsing into arbitrary contentions, additional hypotheses about location entail gratuitous complexity and the sort of arbitrariness Lewis cautions against.

Although Lewis is correct that certain hypotheses about the location of sets are obnoxiously arbitrary, it is conspicuous that our two leading views about the location of sets are omitted from the above passage. Each of transcendent and pervasive platonism has a credible claim to non-arbitrariness as well as the requisite generality to count as fully fledged theory about the location of mathematical reality.

Lewis's remarks above about location are followed in *Parts of Classes* by a potential argument against the view that {Possum} is located where Possum is.[18] Roughly put, if sets are mereological atoms occupying extended regions, then sets would be related to regions in a manner unlike most ordinary objects, which are standardly held to have parts corresponding to the distinct locations they occupy. Lewis is quick to add that, despite being a "peculiar" result, this is a consequence that can be abided. Given our ignorance about the intrinsic nature of sets, including their mereological properties, this result does not violate any especially clearly held commitments. Even so, this line of argument can be re-directed against pervasive platonism: if

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[17] Lewis (1991: 10-13) defends a non-standard conception of the null set as the sum of concrete reality. We set aside complications raised by such a view here.

[18] Lewis (1991: 32).

mathematical reality is omnipresent, wouldn't it have to have distinct parts at any extended region it occupies? In keeping with the standard response offered by proponents of Aristotelian universals, we need not hold that the same principles regarding parthood and location obtain for both physical objects and mathematical ones. So, while mathematical reality might have parts—e.g., Lewis argues that classes are fusions of singletons—its spatiotemporal profile need not determine its mereological structure. Much like the argument regarding shape considered above, pervasive platonists can resist this argument, but we will shortly consider an alternative strategy in Section 5 that avoids it altogether.

Thus far, careful consideration of theoretical virtues and vices provides no clear verdict for or against positing the location, much less omnipresence, of mathematical reality. There is, however, a final axis of complexity to consider when assessing views about the location of mathematical reality: their modal status. Generally speaking, merely contingent theses are weaker than their necessitated counterparts. But, when doing metaphysics, claims of contingency can also raise odd questions for proposed theories. Suppose one endorses transcendent platonism but only as contingent hypothesis. This entails that there are worlds where mathematical entities are indeed located. So what evidence supports belief in their absence of *actual* location? And, what, if anything, could explain this metaphysical difference between possible worlds? Is this an objectionable brute fact that follows from accepting this matter as a contingent one? Somewhat perversely, necessitated versions of transcendent and pervasive platonism might now seem more tractable than their contingent counterparts.

Once we acknowledge modal variation of these theories, rivals to transcendent and pervasive platonism emerge. Consider the following view, which seeks to avoid any sort of arbitrariness: for any way the world could be and any pattern of location that sets might exhibit (e.g., being without location, being pervasive, or even being wholly located in Nebraska), there is a possible world where sets are so located. Such a view concedes the truth of transcendent and pervasive platonism albeit in only some possible worlds. And, while its concession to their coherence might seem initially attractive, this view has explosive metaphysical consequences, multiplying the possible worlds we are required to countenance in a way that seems to secure no significant advantages. While this and related views warrant scrutiny, their ultimate assessment hangs on issues about arbitrariness and theory choice that would take us too far afield. We will therefore remain focused on the comparative merits of transcendent and pervasive platonism, but conclude that tallying up virtues and vices provides us with little direct guidance.

4. The Case for Pervasive Platonism

The preceding section suggests a significant conclusion about the debate over the location of mathematical entities: that the case for transcendent platonism is weaker than standardly assumed. Given that most platonists endorse transcendent platonism, this is somewhat noteworthy—but, absent some positive argument for pervasive platonism, it would seem that the reasonable stance regarding the location of mathematical entities is agnosticism. The task of this section is therefore to consider what can be said in favor of pervasive platonism.

The most direct path for defending pervasive platonism would be to demonstrate that it follows from some more basic commitment of platonism.[19] But the basic commitments of platonism are (1) that mathematical entities exist and (2) that they have the features described in our best mathematical theories. Holding mathematical entities to be omnipresent is of no apparent help in accounting for (2) and could only assist in explaining (1) if we were willing to beg the question against transcendent platonism and hold that existence is impossible without being located. Could there be some *non*-mathematical feature of mathematical reality that pervasive platonism might help us explain? Thus far we have followed Lewis in proceeding with serious caution when it comes to the non-mathematical nature of mathematical reality. There is, however, one non-mathematical feature of mathematical reality that we take to be an implicit component of most forms of platonism: what we will call the *invariability* of mathematics—the fact that mathematical truths hold no matter what.[20]

Platonists can, in principle, reject the invariability of mathematics. One reason for doing so is an empiricist commitment to treating mathematics, mathematical truth, and mathematical ontology in parallel with our attitude toward scientific theories.[21] Another reason for doing so would be a Humean skepticism about the necessary connections between entities that platonism requires—e.g., that the entire stock of natural numbers exists of necessity and that no worlds are populated with only twenty-two of them.[22] Few platonists find much appeal in such views. But, in rejecting them, the challenge of spelling out precisely what the invariability of mathematics consists in looms large. In what follows, we assume platonists accept invariability as a non-mathematical feature of reality.

It is natural to interpret the invariability of mathematics in modal terms: it must be true that seven is a prime number. Put in the language of possible worlds: it is true in all possible worlds that seven is a prime number. This sort of reasoning is sometimes used to motivate the conclusion that mathematical entities are necessary existents. After all, how could it be true that seven is a prime number at all possible worlds unless seven existed at all such worlds? Notice, however, that the invariability of mathematics also admits of a temporal interpretation: it is always true that seven is a prime number. Put in the language of times: it is true at all times that seven is a prime number. No platonist is inclined to think mathematical theories were false last month or will be endangered by the heat death of the universe. So the invariability of mathematics seems to require its permanent as well as necessary truth. And, parallel reasoning

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[19] An alternative strategy seeks to vindicate pervasive platonism by showing it to play a key role in addressing objections to anti-platonist arguments. As best we can tell, pervasive platonists fare no better or worse than transcendent platonists in addressing familiar challenges regarding epistemic access, arbitrariness, or ontological profligacy. The most likely objections that omnipresence might play a role in addressing are those regarding the nature of mathematical explanations of physical phenomena (e.g., worries about "extrinsic explanations" in Field (1980)), but omnipresence alone does not guarantee a significantly different explanatory relationship between mathematical and physical reality.

[20] What distinguishes mathematical truths is a relatively delicate matter—e.g., we exclude claims like there are three more planets than there are moons. We leave open whether invariability might prove mandatory given the nature of mathematical practice, but see Yli-Vakkuri and Hawthorne (2018).

[21] See Colyvan (2001) for discussion.

[22] See Rosen (2002) and Cowling (2017) or discussion.

about modality and time invite the further conclusion that mathematical entities are permanent existents. The invariability of mathematics makes an additional demand once we consider space in addition to time, since there is nowhere one can travel where mathematical theories turn out to be false. When it comes to arithmetic, Nebraska and Neptune are no different. We can therefore interpret the invariability of mathematics as an additional thesis about spatial regions: it is true at all regions that seven is a prime number.

These modal, temporal, and spatial theses seek to capture the invariability of mathematics, and the most straightforward account of the invariability of mathematics accepts each of them in turn. Once suitably by supplemented our best physical theory (i.e., talk of space and time is replaced with spacetime), the result is that invariability of mathematics is not only a modal thesis but a spatiotemporal one: that mathematics is true everywhere—i.e., at all possible worlds and all spatiotemporal regions. So, just as platonists are inclined to hold mathematical entities to be necessary existents, pervasive platonists similarly conclude they are pervasive existents, and their omnipresence provides a concise account of the invariability of mathematics.

The *argument from invariability* raises no shortage of questions and potential objections. For one thing, if this line of argument is extended to logical truths which seem no less invariable, it might seem to yield bizarre consequences. Take a logical truth like "Obama is human or it is not the case that Obama is human." If logical truths are invariably true and invariability requires the omnipresence of their subject matter, we would seem saddled with holding Obama (and presumably any other entity) to be omnipresent as well. This objection obliges the platonist to be more careful when mounting their argument. While mathematical truths and logical truths are equally invariable, the platonist need only take the former to carry significant ontological commitments. Mathematical theories entail, not only that seven is a prime number, but that there are infinitely many natural numbers. It is these quantified claims, unlike the logical truth regarding Obama, that are ontologically committing. So, according to the argument from invariability, the claim that there are prime numbers places invariable demands upon what there is; logical truths of the above sort do not.

This argument from invariability hangs on a range of assumptions about truth and existence. The first of these is that understanding invariability requires that, when considering what's true at a given possible world or a specific spatiotemporal region, we restrict our domain of quantification to all and only those entities that exist at that world or region. So, if mathematical entities do not exist at the relevant world or region, our mathematical theories turn out to be false. When we consider how transcendent platonists ought to respond to the argument from invariability, a natural strategy therefore emerges: to defend the view that, in any relevant sense, mathematical truths obtain everywhere but that mathematical entities need not exist everywhere.

In a postscript to "Counterpart Theory and Quantified Modal Logic," Lewis sets out a view that seems carefully tailored to ensure this result. In addressing complications that stem from his modal realist account of possible worlds, Lewis claims that there are a variety of domains of quantification we might associate with individual possible worlds. There is, however, no uniquely determinate relationship between a world and a domain of quantification. Moreover, the domain of quantification ordinarily associated with the actual world is inclusive of mathematical entities even though they are no part of the actual world. As he says,

> When we evaluate the truth of a quantified sentence, we usually restrict the domain and quantify over less than all there is. If we evaluate a quantification at a world, we will normally omit many things not in that world… But we will not omit the numbers or some of the other sets. Let us say that an individual exists *from the standpoint of a world* iff it belongs to the least restricted domain that is normally—modal metaphysics being deemed abnormal—appropriate in evaluating the truth at the world of quantifications…There will be many sets that exist even from the standpoint of all worlds, for instance the numbers. (Lewis (1983: 40)).

Setting aside Lewis' distinctive commitment to modal realism, this is a tempting strategy for the transcendent platonist to endorse. According to this strategy, mathematical claims are true at every possible world in any ordinary sense and this is because, ordinarily speaking, our domain of quantification includes certain non-worldly entities like numbers. Similarly, mathematical claims are true at any place and at any time in any ordinary sense because, ordinarily speaking, when we restrict our attention to that place or that time, we nevertheless include mathematical entities which exist merely from the standpoint of the world.

If one takes the invariability of mathematics to be a primarily linguistic fact—roughly, that, in ordinary contexts, our quantifiers should be interpreted as inclusive of mathematical reality—we should be entirely satisfied with the Lewisian approach. But the invariability of mathematics is a commitment of platonism that emerges only in what Lewis would describe as "abnormal" contexts of mathematical ontology—contexts where we seem to be acutely aware of the peculiarities of ontology and sensitive to the delicacy of domain restriction. Consider, for instance, that when engaged in the ontology of time, the would-be eternalist takes herself to be perfectly capable of restricting her quantification to all and only presently existing things. We take it that this would-be eternalist, when restricting her quantification in this way, is not at all inclined to now deny that seven is a prime number. And the simple account of invariability given by the pervasive platonist can explain this without positing any linguistic incompetence on behalf of the would-be eternalist. It is not that she has errantly included some non-spatiotemporal entities into her domain of quantification alongside the presently existing ones, it is that mathematical reality—unlike dinosaurs and moonbases—is presently existing, too.

There is more to be said about competing approaches for upholding invariability and especially about heterodox views that would argue against taking mathematics to be a worldly matter true even at a single possible world. But, instead of cataloguing views which deny that invariability requires existence at possible worlds and spatiotemporal regions, we will explore a further complication about what existence relative to these domains might require.

5. Existence and Location for Pervasive Platonists

For the argument from invariability to succeed, the invariability of mathematics must entail that mathematical reality is included in the domain of quantification associated with each possible and spatiotemporal region. On the basis of this fact about domain inclusion, the pervasive platonist holds omnipresence of mathematical reality to follow immediately. But this inference hinges upon a further thesis about how existence at a world or a spatiotemporal region relates to location: that the existence of mathematical entities at a world or spatiotemporal region entails that mathematical entities occupy the relevant world or spatiotemporal region.

It is plausible enough that occupying a world is a sufficient condition for existing at that world. But there are a range of views in modal and temporal ontology that deny entities must occupy some region at a world or at a time in order to exist at the relevant time or world. (Here, and in what follows, we take "region" to be neutral between any kind of spatial, temporal, spatiotemporal, or other sort of region unless noted.) The most familiar of these views is the necessitism defended in Williamson (2010) and elsewhere, according to which the domain of quantification does not vary from world to world, since, at each world, every possible entity exists even though many of them occupy no region at all. A similar view, permanentism, holds that entities—all of them, in fact—exist at all times and that many entities exist at times even without occupying a region at that time. Setting aside the merits of these particular views, they illustrate the coherence of rejecting the occupation requirement, according to which entities must occupy a region at a time or at a world in order to exist at the relevant time or world.

As we have seen, for the argument from invariability to succeed, we must have license to infer that mathematical reality is located at every possible world and spatiotemporal region precisely because mathematical reality exists at every possible and every spatiotemporal region. But the coherence of views like necessitism and permanentism reveals that this assumption is contestable. The coherence of these views is not itself a reason to reject this assumption; however, in addition to being an additional burden the proponent of this argument must discharged, it is a choice-point that divides pervasive platonists into two competing camps.

Throughout the preceding, we have adopted a location-based conception of omnipresence which requires omnipresent entities to be wholly located at each and every spatiotemporal region. But our pre-theoretic thought and talk about omnipresence is far murkier than this conception suggests. Even if omnipresence requires that an entity is everywhere, what it is to "be everywhere" admits of a wide range of precisifications. One of these can be formulated in terms of existential notions rather than in terms of location. According to what Cowling and Cray (2016) call the "Existential View" an entity x is omnipresent if and only if for any region r, x exists at r. This Existential View takes omnipresent entities to exist at every region but it is silent on whether such entities occupy all regions or, in fact, whether they are located anywhere at all. In metaphysical analogy with the necessitist who holds the Eiffel Tower to exist at every possible world regardless of whether it occupies a region, the Existential View of omnipresence holds an omnipresent entity to exist at every spatiotemporal region regardless of whether it occupies one.[23]

Cowling and Cray (2017) argue that the Existential View enjoys certain advantages as a tool for making sense of claims regarding omnipresent entities. Given our present focus, the distinction between the existential conception and the location-based conception yields two different versions of pervasive platonism. According to *locational pervasive platonism* (LPP),

[23] On necessitism, see Williamson (2010).

mathematical reality is wholly located at each and every spatiotemporal region.[24] According to *existential pervasive platonism* (EPP), mathematical reality exists at each and every region but is not spatiotemporally located. The truth of EPP depends upon the possibility that entities can exist at regions without occupying them. In contrast, if the argument from invariability is to establish LPP, it can only do so if entities must occupy any regions at which they exist. Each of these views therefore comes with built-in challenges. We will conclude, however, by noting that for those sympathetic to pervasive platonism, EPP enjoys three significant advantages.

First, two of the arguments we considered for transcendent platonism revolved around the difficulties that arise in holding mathematical reality to be located. The first sought to show that this led to the implausible view that mathematical reality instantiates shape properties. The second sought to show mathematical reality would have to violate familiar principles about parthood and location by occupying extended regions without thereby having parts at them. Each of these arguments can be set aside by adopting EPP instead of LPP, since existence at a region need not require occupation and, in turn, the instantiation of location or shape properties.

Second, we have assumed throughout that reality is spatiotemporal in nature rather than being a classical world with distinct spatial and temporal dimensions. We have also assumed the truth of substantivalism. There is, however, good reason to believe that worlds with distinct spatial and temporal dimensions are possible and that there might be worlds that are merely analogously spatiotemporal.[25] There is also good reason to think that substantivalism is a contingent hypothesis and that supersubstantivalism and relationism are true at some worlds. A theory about the nature of mathematical entities that ties invariability to facts about the location relation they bear to spatiotemporal regions is therefore oddly tethered to the contingent physical structure of reality. Faced with this awkward result, omnipresence could be understood as a disjunctive thesis that mathematical reality is wholly located at every region in substantivalist worlds of spacetime or stands in such-and-such relations in relationalist worlds of space and time or such-and-such relations in supersubstantivalism worlds and so on. But such a view is dubiously complex and perhaps resists complete formulation. In contrast, EPP has no essential dependence on the ideology of location and the notion of spatial or spatiotmporal region. (Recall that above "region" can be understood as neutral between these and other notions.) It can hold in full generality in a way that makes the presence of mathematical reality independent of the contingency of these physical facts. In this way, EPP proves to be more attractive than LPP especially since each purports to describe the features of mathematical reality which is characterized by its invariability.

Third, only EPP permits a kind of armistice with transcendent platonists. If we interpret transcendent platonism as we have done throughout the preceding discussion, it is the denial that mathematical reality is located anywhere at all. And this thesis is perfectly compatible with mathematical omnipresence as understood by EPP. Surprisingly, then, EPP secures substantial agreement with the transcendent platonist by distinguishing between two importantly different

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[24] LPP entails that mathematical reality exists at each and every region in virtue of being so located. Given our definition of EPP in terms that specifically preclude location, it is false that LPP entails EPP.

[25] See Lewis (1986: 69-81)

ways in which entities might be related to worlds and spatiotemporal regions. The transcendent platonist might be quick to reply that their thesis should now be similarly reinterpreted: it's not merely that mathematical reality is not located, but, rather, there are no world or regions at which mathematical entities exist. But, while invariability is a claim about mathematical reality platonists are liable to be sympathetic toward (and one we have assumed they accept), the corresponding motivation for this existential reinterpretation of transcendent platonism would have to be a much stranger and more controversial feature—something like the *elusiveness* of mathematical reality, which precludes mathematical entities from existing at times or worlds. Better, it seems, for pervasive platonists to avoid the perils of locating mathematical reality and help themselves to the simplest account of the invariability of mathematics.

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