Approximating Electoral Control Problems

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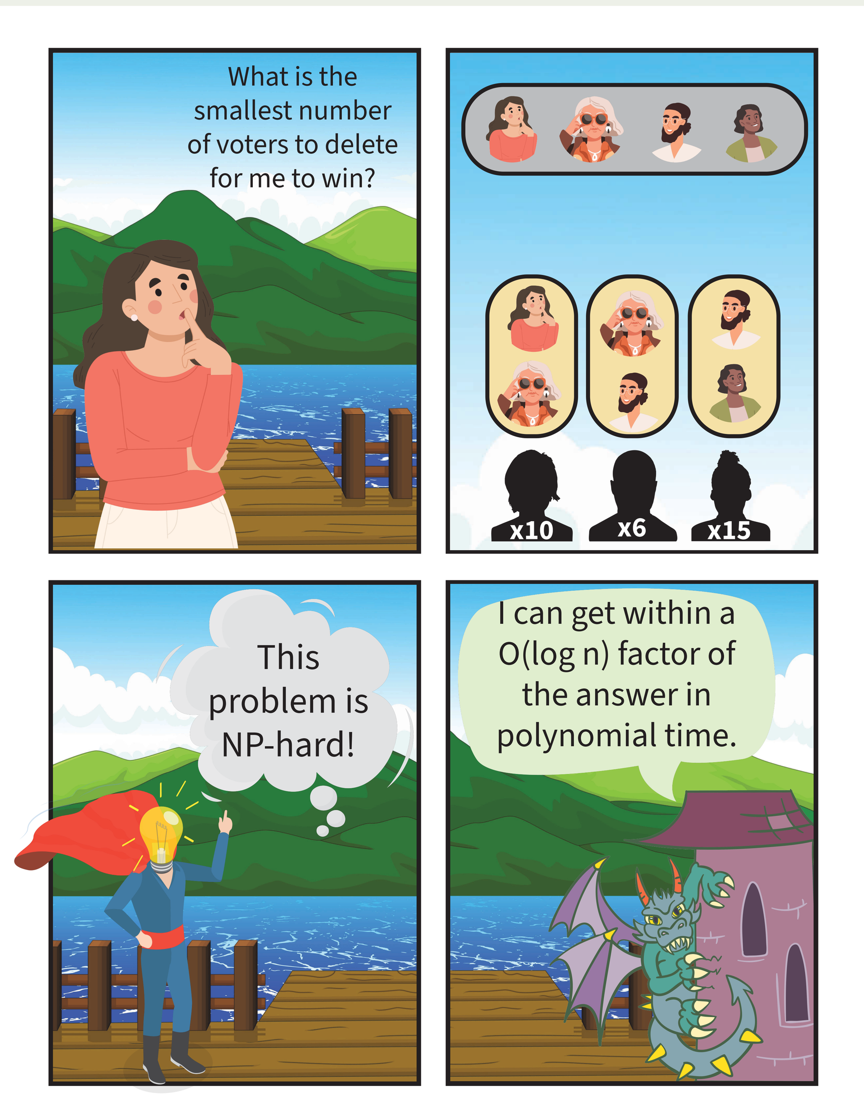
Overview

- Electoral control: one of the most studied forms of electoral attacks, in which an entity alters the structure of an election to yield a preferred outcome.
- Research has focused on mainly decision-complexity results.
- Approximations have received little attention, despite its prevalence in the study of other forms of electoral attacks, such as manipulation and bribery.
- Early work established preliminary results with respect to popular voting rules such as plurality, approval, and Condorcet.

Model

- An election is a pair (C, V), where C is a set of candidates and V is a list of votes over C.
- A voting rule maps an election (C, V) to a winner set.
- Constructive control: make a candidate a unique winner.
- Destructive control: prevent a candidate from being a unique winner.
- Standard control problems: constructive/destructive control by adding/deleting/partitioning candidates/voters.
- Optimization algorithms seek to minimize some property (called a *measure*) of a solution, e.g., computing the smallest number of voters to delete.

An Example under Approval



Objectives

- Determine the landscape of approximability for the standard electoral control problems with respect to:
  - Plurality: a candidate receives one point for each vote that ranks them first.
  - Approval: a candidate receives one point for each vote that approves them.
  - Condorcet: a winner pairwise defeats every other candidate in a majority vote.
- Explore previously-identified barriers to studying approximability in partition-based problems:
  - How to define a suitable measure?
  - How to design inapproximability results due to lack of restrictions on partitions?

Highlights of Results

- For every partition-based control problem $\mathcal{T}$ (regardless of the underlying voting rule), if the decision problem for $\mathcal{T}$ is NP-complete, then the corresponding optimization problem (regardless of the measure used) has no approximation algorithm, unless $\mathrm{P} = \mathrm{NP}$.
- Under approval both constructive control by adding and deleting voters are $\mathrm{Log\text{-}APX}$-complete.
- Voiced rules: Constructive control by deleting candidates is in $\mathrm{Poly\text{-}APX}$, using a simple greedy algorithm.
  - Under the "Dense versus Random" Conjecture [3], the approximation lower bound for plurality is in $\Omega(m^{1/4})$.
- All other problems we studied: if the decision problem is NP-complete, then the optimization problem has no approximation algorithm, unless $\mathrm{P} = \mathrm{NP}$.
  - In each case, we showed that just computing a solution—even when no bound on the solution is given—is NP-hard.
- Our results hold even if votes are weighted.

Conclusions

- Many control problems remain hard in the optimization model.
- Except for plurality-CCDC (constructive control by deleting candidates under plurality), we've completely determined the approximability of the standard control problems under plurality, approval, and Condorcet.
- We have circumvented the presumed barriers to studying partitioning in this setting by using a simple and easy-to-state argument.

Future Work

- Extend this work to account for other voting rules.
- Study approximations for control problems in P. Those may not have polynomial-time computable optimization (or even search [1]) algorithms [4].
- Improve the lower bound the approximation ratio for plurality-CCDC (constructive control by deleting candidates under plurality).
- Determine general conditions on voting rules or on control problems that imply the existence of approximation algorithms. Even better would be to give complete characterizations.

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Table of results

Table 1. Our results are boldfaced. Others are from [2]. A "-" means the decision problem is in P.

Control by	Plurality		Condorcet		Approval	
	Construct.	Destruct.	Construct.	Destruct.	Construct.	Destruct.
Adding Candidates	Inapprox.	Inapprox.	-	-	-	-
Deleting Candidates	**Poly-APX**/$\Omega(m^{1/4})$	**Inapprox.**	-	-	-	-
Partition of Candidates	TE: **Inapprox.** TP: **Inapprox.**	TE: **Inapprox.** TP: **Inapprox.**	-	-	TE: - TP: -	TE: - TP: -
Run-off Partition of Candidates	TE: **Inapprox.** TP: **Inapprox.**	TE: **Inapprox.** TP: **Inapprox.**	-	-	TE: - TP: -	TE: - TP: -
Adding Voters	-	-	**Inapprox.**	-	**Log-APX-Complete**	-
Deleting Voters	-	-	**Inapprox.**	-	**Log-APX-Complete**	-
Partition of Voters	TE: - TP: **Inapprox.**	TE: - TP: **Inapprox.**	**Inapprox.**	-	TE: **Inapprox.** TP: **Inapprox.**	TE: - TP: -

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