Applying Genetic Algorithms to Ramsey Theory

David White
Wesleyan University

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1. Basic Graph Theory and graph coloring

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2. Pigeonhole Principle
3. Definition and examples of Ramsey Numbers

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4. Generalizing Ramsey numbers

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4. Generalizing Ramsey numbers
5. Ramsey's Theorem
6. Some bounds on Ramsey numbers
7. One further generalization

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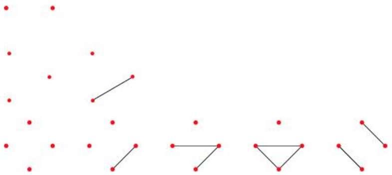

Figure: A graph

Complete Graphs

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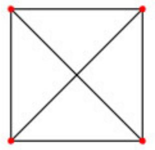
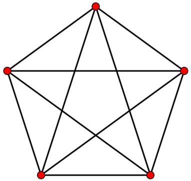


K_4



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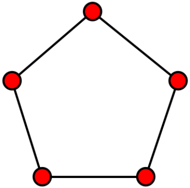

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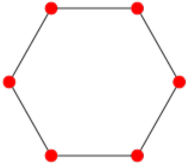

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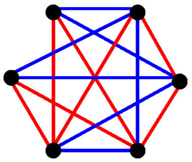

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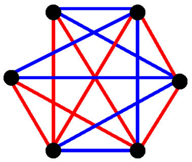

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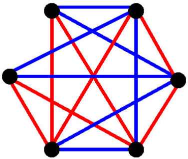

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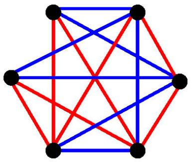

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We will be interested in colorings which avoid monochromatic subgraphs. This has no red triangle and no blue triangle, but last edge will force a monochromatic triangle.

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The notion of placing pigeons into 2 holes is exactly the same as 2-coloring the pigeons.

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Generally: what is the smallest model guaranteed to contain the submodel I desire?

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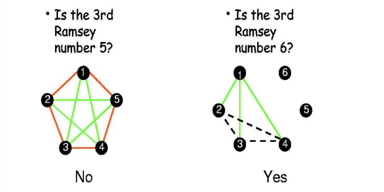

Figure: $R(3) \geq 6$

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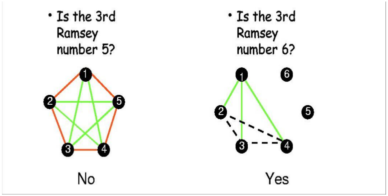

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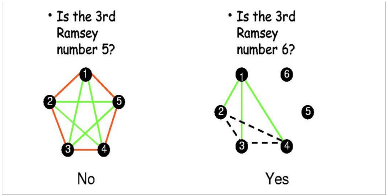

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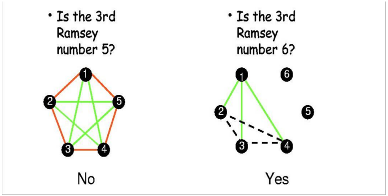

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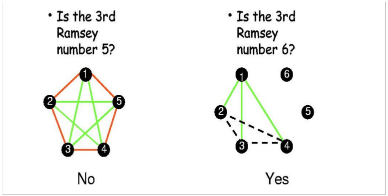

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If any edge is green then we have a green triangle.
So all of these edges must be red, giving a red triangle. □

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There are $\binom{43}{2}$ edges and each has two choices, so number of colorings is $2^{\binom{43}{2}} \approx 2^{1000}$. This is a HARD problem.

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Constructive methods like this can give polynomial lower bounds of any fixed degree, but nothing reaching c^n.

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We will focus on constructing lower bound examples.

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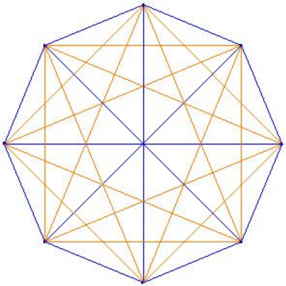

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$R(T_m, K_n) = (m - 1)(n - 1) + 1$ for any tree T_m

Examples

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David White Wesleyan University

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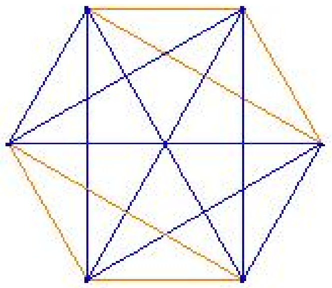

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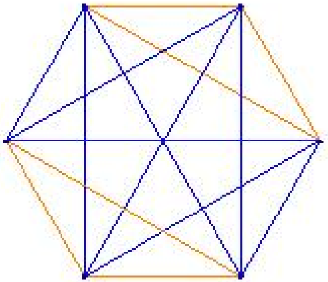

$$\begin{bmatrix} & y & y & b & b & b \\ y & & y & b & b & b \\ y & y & & b & b & b \\ b & b & b & & y & y \\ b & b & b & y & & y \\ b & b & b & y & y & \end{bmatrix}$$

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A lookup table is used to get values of i and j from given edge $e = (i, j)$.

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Example: $p_1 = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$ and $p_2 = (4\ 1\ 2\ 8\ 7\ 6\ 9\ 3\ 5)$

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Selection and Mutation

Tournament Selection:

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David White
Wesleyan University

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Fitness Function

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Checking for other G_a is similar and easy.

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initialize population and set fitness[i] =fv(i) for all i
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Statistics

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Mutation rate on all good runs was 0.001, the lowest possible. So again, algorithms is mostly exploitation.

Rao matched known bounds for $R(C_4, C_4, C_4)$, $R(C_4, C_4, K_3)$, $R(C_4, K_3, K_3)$, and $R(C_5, C_5, C_5)$. He used special seed values for these based on prior results.

He found new bounds on never before investigated numbers: $R(C_4, C_4, K_3, K_3) \geq 25$ and $R(C_5, C_4, K_3) \geq 13$. How much does this matter?

He found numerous different colorings to prove these, but one coloring suffices for a proof.

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- Ask about Ramsey numbers of directed graphs or hypergraphs.

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Use a hierarchical GA: solve smaller instances of the problem and then combine solutions. If $R(G_1, G_2) = n$ avoid an uncolored K_n when you have only 2 colors left.

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