Numerical Tools for Describing Musical Compositions

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ABSTRACT

Our goal for this paper is to apply the methodologies of statistical mechanics to develop flexible tools for use in computational musicology. Classical statistical mechanics is the study of extremely information-rich particle systems. Large quantities of information describing the position and velocity of each particle may be compressed into a few descriptive variables such as temperature and pressure. These variables are easily perceived by an observer. In the same way, we will distill information from the scores of musical compositions, producing new macroscopic quantities that describe perceptible features of the music.

1. INTRODUCTION

1.1 Statistical Mechanics

In the terminology of thermodynamics, a system is the subset of the universe with which we are concerning ourselves (the remainder of the universe is the *environment*). Thermodynamics involves observable variables exhibited by a system, which are called *observables*. Examples include temperature, pressure, and volume. These observables provide an easy means of comparing different systems and of quantifying perceived experiences. That is, a system feels hot because its temperature is higher than that of the observer's hand. A system has high pressure because it compresses objects which are placed within it.

Thermodynamics was developed before the concept of fundamental particles was widely accepted. For early scholars of thermodynamics, temperature and pressure were intrinsic, measurable properties of a system. Statistical mechanics came about after the discovery that systems were in fact collections of tiny fundamental building blocks. Thus, a gas is a collection of particles which interact in a specific way to exhibit the properties we associate with a gas. In this way, our observables are, in fact, functions of more fundamental variables characteristic of the particles.

In the model of statistical mechanics, a system containing a classical gas can no longer be unambiguously described by temperature, pressure, and volume. Even this very simple system contains a huge amount of information. In order to perfectly describe such a system, one would need to compile a list of the position and velocity of every individual particle present in the system at every instant of time. This is called the microstate of the system. However, the observables seen by a human specify the *state* of the system. Many microstates can exhibit the same state when observed on a higher scale. Statistical mechanics allows us to derive functions of the many independent variables, typically as averages over the particles, over time, or over both. These functions distill the microstate information to depict the large-scale state of the system. Examples of these functions correspond with the human experiences of temperature or pressure.

This idea of distilling microstate information into macrostate information which is relevant to human observations will prove to be key to the computational musicology we describe in this paper. Before we implement these ideas, it is necessary to discuss a few relevant concepts of musical scores and music theory.

1.2 Musical Background

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For the purposes of this paper, we are only concerned with music composed in the western classical tradition.

A piece of music comprises a finite collection of individual notes. Each note exhibits a certain pitch, representing how high or low the note sounds to an observer. This is physically represented by the frequency of the sound (measured in Hertz (Hz)). A perceptual phenomenon, believed to be universal to human experience, is that of *pitch class* or *octave equivalence*. Consider the pitch with frequency 440 Hz. We find that a pitch with frequency 220 Hz — exactly half of the frequency and one octave below — is perceived as being qualitatively similar to the 440 Hz pitch. We say that all pitches which possess this same quality are in the "A" pitch class.

How many pitch classes are there? Historically, this has been the subject of much debate [1]. In the past, unique tunings have been constructed to maintain the ratio of a perfect fifth $(3:2)$ as well as possible. In modern Western music, the equal temperament system divides the frequency range between one A (say 440 Hz) and the next A (at 880 Hz) into twelve uniform intervals, resulting in twelve unique pitch classes. Each pitch in this *octave* is separated by one semitone (with two semitones being designated a step). The mathematical symmetry of equal temperament disrupts the small-integer frequency ratios of the perfect intervals: the fourth (4:3) and the fifth (3:2), leading to the aforementioned debate. For our purposes, equal temperament is to be preferred because a musical score in equal temperament designates unambiguously the pitch of each note it contains.

Other aspects of music manifest themselves in the presence of more than one note. The time interval between the onsets of separate notes creates the perception of rhythm (how the notes of the piece are positioned in time). The unit of time for music is defined as the *beat*. The *meter* determines the pattern of accented beats. The *time signature* of a piece defines the unit of a beat and the pattern of accents. The most common meters are $\frac{4}{4}$ time and $\frac{3}{4}$ time. In $\frac{4}{4}$ time, every fourth beat is accented and we group the beats of the song in four beat long *measures*. A classic example is "Twinkle, Twinkle Little Star". A $\frac{3}{4}$ time piece accents every third beat and the piece is divided into three beat long measures. This is the time signature used by most waltzes. A famous example is "Amazing Grace".

In addition to the temporal relation between two notes, they are also separated by an interval in the space of pitches, which we describe simply by the number of semitones between the two pitches. Some common intervals are expressed in Table 1.

Half step	One semitone
Step	Two semitones
Minor third	Three semitones
Major third	Four semitones
Perfect fifth	Seven semitones
Octave	Twelve semitones

Table 1: Common intervals in number of semitones.

Furthermore, gathering more than two pitches together creates the concept of harmony. Three pitches form a triad. A triad constructed by a major third followed by a minor third is called a major triad. A minor triad is constructed with a minor third stacked below a major third.

The set of twelve pitch class representatives comprises an octave. This comes from the concept of a *scale*: a sequence of pitch classes that begins and ends with the same pitch class. A scale may begin on any of the twelve pitch classes. There are several flavors of scales which are determined by the sequence of intervals used to construct the notes of the scale. For example, the major scale is constructed with the following intervals: step, step, semitone, step, step, step, semitone.

Observe that this sums to twelve total semitones: the scale returned to the pitch class at which it began. Note also that the most commonly heard scale contains eight notes (hence the term "octave"). Like triads, scales can be major or minor. A composer will frequently designate a *key signature* for a piece, indicating the scale from which the majority of the pitches will come. If a piece is written in the key of A major, we call the A major chord the home key (I). Generally, a piece in a major key has a brighter or happier sound than a piece in a minor key, which often sounds somber or dramatic.

2. STATISTICAL MECHANICS APPLIED TO MUSIC

Recall from the statistical mechanics discussion, we as human observers perceive temperature and pressure. As physicists, our understanding of particle dynamics allowed us to explain the observed variables as functions of more basic variables.

For the musical analysis we describe here, we work in reverse. Analogous to the particle microstates of position and velocity are the pitch and rhythm information of each note in a piece of music. This is provided to us in the form of the musical score as written by the composer. We will endeavor to distill the musical microstate information into functions that describe broader features of the piece of music. We call these variables *hearables* in analogy to the observables of statistical mechanics. We propose that these hearables model the perception of a listener.

2.1 Basic musical variables

For this study, we focus on musical compositions which are encoded into the MIDI format [9]. The MIDI format [8] was invented in 1983 as a way to store musical data taken from electronic keyboards. It is a widely accepted protocol and contains data for all of the variables we wished to study. MIDI files take the form of long strings of hexadecimal numbers specifying everything from the pitch, volume, and onset time of each note in the piece to the time signature and comments like copyright data.

For the purposes of this study, we define three features of individual notes that we consider as basic variables. First is the pitch of the note — the frequency of the note's sound wave — rounded to the nearest pitch class in the western 12-note scale. In MIDI, the note known as middle C on the piano keyboard (256 Hz) is given a pitch value of 60. The unit of measurement is a semitone, so 61 corresponds to the C$^\sharp$ above middle C and 72 corresponds to the C an octave above middle C. In general the MIDI note number p corresponds to the frequency f of the pitch according the formula

$$p = 69 + 12 \log_2 \frac{f}{440}.$$

Second, the MIDI onset time provides a rhythmic variable. Some manipulation of MIDI code allows us to represent the onset time of each note in terms of beats instead of milliseconds. A note that begins on the first beat of the first measure of the piece has an onset time of 1. A note falling on the second half of beat two in measure one has an onset time of 2.5. In $\frac{4}{4}$ time, beat one in measure 2 has onset time 5, while in $\frac{3}{4}$ time, beat one in measure 2 has onset time 4, and so on. We do not explicitly consider the duration of notes, and only concern ourselves with the time at which notes begin, as this contains the most pertinent rhythmic information.

Third, for much of what follows, it is useful to distinguish notes played by different instruments, or *voices*. MIDI format makes this very simple as one of its stored variables designates the channel of each note. That is, which instrument played which note. In some cases, such as keyboard works for which a single instrument effectively plays several parts simultaneously, we simply designate all notes of pitch higher than middle C (60) as being one voice and all notes below middle C being another voice.

2.2 Proposed Hearables

In the same way that the thermodynamic variables of temperature and pressure correspond to features of a system that are relevant and easily observed by a human observer, our hearables ought to be easily understandable and relatable to a listener. In this vein, we have considered our own listening habits and consulted with avid audiophiles and professors of music theory to discover what it is that allows us to classifies musical pieces. In this work, we design and implement algorithms that compute these hearables from MIDI files. The result is a suite of computer software that can detect the features of music that listeners deem most representative of a musical style, and perhaps can detect subtler features not obvious to a casual or focused human listener.

One easily quantifiable idea is the complexity of a piece. Different eras of music are known for intricate rhythms, harmonies, and melodies, while other eras are more stark and minimalistic. In what follows, we quantify complexity as entropy.

Another hearable relates to the idea of signature patterns. Just as many literary authors have favorite turns of phrase or characteristic terminology, composers exhibit signature ideas in several domains of musical composition. Within a piece, a composer might repeatedly develop a rhythmic or melodic idea. In a broader sense, a composer might have a characteristic rhythm, melody, or harmonic progression that one would expect to find in many pieces by this composer. A listener who hears this "Mozart-like" rhythmic pattern can guess that the piece was composed by Mozart.

Finally, the harmonic structure of a piece is indicative of its era. Generally, composers grew more adventurous with harmony over the centuries. Very early compositions invoke only the harmonies of the home key and very near neighbor keys. On the other end of the spectrum, modern composers experiment with atonal music which does not conform to any well-defined key. We visually present the distribution of harmonies in a given piece or collection of pieces as a *harmonic landscape*.

3. FLEXIBLE TOOLS FOR MUSICAL ANALYSIS

3.1 Complexity

Complexity can be quantified as a form of entropy. Entropy, in the sense of statistical mechanics, might be said to characterize the unpredictability of a system. A piece of music that is entirely predictable is, by definition, not very complex. Consider a piece consisting solely of a single pitch played at a constant rate. This is perfectly predictable (low entropy) and also very simple (low complexity).

Given a probability distribution $P(x_i)$, we can calculate the entropy of the distribution with the following equation:

$$S = -\sum_{i=1}^{n} P(x_i)\log(P(x_i)).$$

In order to evaluate the entropy or complexity of a piece, we only need to generate distributions which represent the piece as heard by an observer. As suggested by Madsen and Widmer [5], we separately evaluate the melodic and rhythmic complexity of a single voice in a piece and average them to find a total entropy for that voice. We give the rhythmic and melodic entropy equal weight. Melodic entropy is evaluated in two ways, by pitch class and by intervals, which we again weight equally:

$$S_{musical} = \frac{1}{4}(S_m + S_{int}) + \frac{1}{2}(S_r).$$

Our probability distribution for rhythm is the collection of inter-onset times (that is, the time elapsed between two adjacent notes in a single voice of a piece):

$$P_r(x_i) = \frac{\text{number of occurrences of inter-onset time } x_i}{\text{number of notes in the piece} - 1}$$

$$S_r = -\sum_{i=1}^{n} P_r(x_i)\log(P_r(x_i)).$$

In this case, minimum entropy occurs when notes are played at some constant rate, independent of pitch.

For the melodic entropy, we used two distributions to capture melodic complexity. One is the distribution of pitch classes of a voice:

$$P_m(x_i) = \frac{\text{number of occurrences of pitch class } x_i}{\text{number of notes in the piece}}$$

$$S_m = -\sum_{i=1}^{n} P_m(x_i)\log(P_m(x_i)).$$

In this case, minimum entropy is given by the same pitch repeated constantly.

Unfortunately, a simple musical scale would achieve maximum entropy with this measure (each note is a different pitch). Thus we also use the distribution of intervals between two adjacent notes as an indicator of melodic entropy:

$$P_{int}(x_i) = \frac{\text{number of occurrences of interval } x_i}{\text{number of notes in the piece} - 1}$$

$$S_{int} = -\sum_{i=1}^{n} P_{int}(x_i)\log(P_{int}(x_i)).$$

In this case, a simple scale possesses minimum entropy.

We average these two quantities for a generalized melodic entropy. Thus, we calculate the rhythmic and melodic entropy of a voice in a piece and then average these quantities to find the entropy of that voice. For pieces comprising more than one voice, we average over the n voices to represent the total complexity of the entire piece:

$$S_{total} = \frac{1}{n}\sum_{i=1}^{n} S_{musical}.$$

We did not implement a measure for harmonic entropy, nor did we consider harmonies in our calculation of the net complexity of a voice or piece.

3.2 Signature Patterns

Within a piece of music, rhythmic, melodic, and harmonic ideas are repeated and developed in interesting ways. An excellent example can be found in Beethoven's Fifth Symphony, in which the iconic "Bum-bum-bum BUM" rhythm is repeated hundreds of times, with different pitches, throughout the first movement of the piece. We have developed a program to search MIDI files for these repeated patterns.

At the lowest level, the program can be provided a target rhythmic or intervallic pattern or a target harmonic progression. We search the given piece for examples where the target pattern occurs. With an additional loop, the program uses each measure of each voice of a piece as a target and searches for duplicates located in any later measure in any voice. Any rhythmic or intervallic pattern that occurs two or more times in a piece is recorded as a pattern for that piece. An additional input parameter is a tolerance, which allows for pieces with flexible rhythm (such as jazz improvisation) to still present patterns. The program can also be instructed to consider only patterns that are shorter or longer than a single measure. Finally, the melody pattern finder can be altered to consider diatonic (within the scale) motion as identical even when the exact intervals differ slightly. For example, in C major, C-D-E would be written as "+2, +2" whereas D-E-F would be "+2, +1" although the two patterns are diatonically identical.

The output for these pattern finder algorithms comes in two parts. The programs output the patterns in an easily-read format, and also provide a tally table indicating the measures in which each pattern appears. Rhythmic patterns are written as a vector of the onset times of each note in the pattern. Thus, Beethoven's well known pattern would read in our format "1.5, 2, 2.5, 3."

13.0000	0	1.5000	2.0000	2.5000	3.0000	0
14.0000	0	1.5000	2.0000	2.5000	3.0000	3.5000
15.0000	0	1.5000	2.0000	2.5000	3.0000	4.5000
16.0000	0	2.0000	0	0	0	0
17.0000	0	2.0000	2.5000	3.0000	0	0
18.0000	0	2.0000	2.5000	3.0000	3.5000	4.0000
19.0000	0	2.0000	2.5000	3.0000	4.0000	4.5000
20.0000	0	2.0000	3.0000	0	0	0

Figure 1: Excerpt from violin part for first movement of Beethoven's Fifth Symphony and example output from rhythm pattern finder program applied to this piece. Note the iconic rhythm in line 13.

The example output shown in Figure 1 was generated by searching for four-beat rhythmic patterns in Beethoven's Fifth Symphony. Observe that line thirteen contains the iconic "bum-bum-bum BUM" rhythm. An additional output catalogs the measures in which each of these rhythmic patterns appears (that is, if we look at line thirteen of the other output data structure, we see which measures contained this rhythm). It should be noted that the line number (13 here) merely indicates that this is the 13th in an unordered list of identified patterns.

Once these within-piece patterns have been found for a number of pieces by the same composer, we can search for patterns which occur in two or more separate pieces - signature patterns. When given a collection of pieces by the same composer, we locate the patterns found within each individual piece. Comparing these located patterns, we identify the patterns that appear in more than one piece. These signature programs again provide two outputs. Each signature is written in an easily-read format along with a count of the separate pieces exhibiting the associated pattern. The signature rhythm program groups pieces in triple time (three or six beats per measure) separately from pieces in duple time (the number of beats in a measure is a power of two) as the rhythmic language differs so greatly between the two. Similarly, for harmonic signatures, pieces in major and minor keys are treated separately.

4	100	−2	−1	1	100	100	100	100
3	100	−2	0	−2	100	100	100	100
3	100	−1	1	−1	100	100	100	100
3	100	0	2	2	100	100	100	100
3	100	1	−1	1	2	100	100	100
3	100	1	2	−2	100	100	100	100
2	100	−12	0	0	0	0	100	100
2	100	−3	−4	7	100	100	100	100
2	100	−2	−2	−1	100	100	100	100
2	100	−2	−2	0	100	100	100	100

Figure 2: Example output from signature interval program. The leftmost column indicates the number pieces in which the pattern is found. Columns three through the end depict the pattern of intervals. The 100s are placeholders.

The example output shown in Figure 2 was generated by searching for interval patterns throughout the complete collection of Mozart's string quartets. Observe that the pattern "$-2, -1, 1$" occurs in four separate pieces in the collection. Similarly, a descending octave (-12) followed by five repeated notes (0) occurs in two of the string quartets.

3.3 Harmonic Landscape

From the macroscopic pitch data, we assign a chord of best fit to a collection of notes as an approximation of harmony. Krumhansl and Kessler [3], in their perceptual psychology study, cataloged how well each of the twelve pitch classes fit within each of the 24 major and minor chords. This generated 24 distributions representing perceived goodness-of-fit of each pitch class within each chord. Given a collection of notes, we construct a distribution of pitch classes. By comparing this distribution to the self organizing map data of Krumhansl and Toivenen [4], we identify the chord that fits best. The program has an adaptive feature that can identify the chord of best fit at the finest appropriate temporal scale — the pitches within one beat, a collection of beats, etc.

Once this has been done, we compute the distribution of harmonies within the twenty-four major and minor keys. This data is illustratively pre-

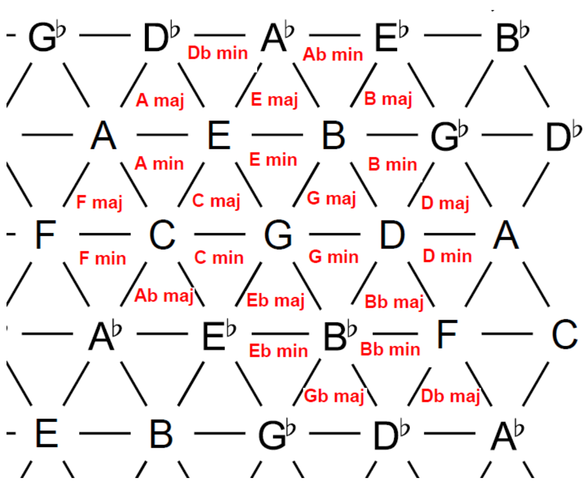

Figure 3: Major and minor keys arranged in a two dimensional Tonnetz.

sented in a two dimensional histogram with the twenty-four major and minor keys arranged in accordance with the two-dimensional Tonnetz [2], a map of keys shown in Figure 3 in which keys with close harmonic relationship are also nearby geometrically. In this way, keys that are closely related harmonically are near to each other in our histogram. When considering collections of pieces, we transpose every piece into the same key — we replace the Tonnetz C major and c minor with the harmonic symbols I and i (that is the major chord of the first note of the key, the minor chord of the first note of the key).

Since the original key of the piece is unimportant in this context, we can aggregate harmonic counts between pieces. Thus, given a collection of pieces by a composer, we generate normalized aggregate histograms presenting the frequency of each harmony relative to the given home key. We treat keys in major and minor keys separately to accommodate the differing harmonic languages.

4. APPLICATIONS

4.1 Complexity

As an application of the usefulness of the complexity measure, we explore how complexity varies over a period of time by finding the average complexity of various composers from the Baroque and Classical periods. From the early Baroque period, Jean-Philippe Rameau (1683 – 1764) exhibits a complexity of 2.2950 (standard deviation 0.2040) and Francois Couperin (1668 – 1733) shows a complexity of 2.2980 (standard deviation 0.1688). We see that these two composers present nearly identical complexities. These complexities are the highest of all the composers we investigated, as might be expected: Baroque music is characterized by complex ornamentation, such as trills, which would serve to artificially increase the rhythmic and melodic entropy of a piece.

From the late Baroque period, George Frideric Handel (1685 – 1759) presents a mean complexity of 1.8367 (standard deviation 0.1858), while Johann Sebastian Bach (1685 – 1750) exhibits a complexity of 1.7073 (standard deviation 0.1876). We see that, for these slightly later composers, the complexity is significantly lower than the early Baroque composers. One wonders if this is indicative of a rapid mentality shift with regards to complexity over the course of the Baroque period, or if this discrepancy can be explained by the geographic differences of the composers (Rameau and Couperin were French, while Handel and Bach were German). This low complexity is consistent with a characterization of Bach's music as being largely scalewise in motion, often with simple rhythmic organization.

Finally, we have the string quartets of Classical composers Joseph Haydn (1732 – 1809) and Wolfgang Amadeus Mozart (1756 – 1791). Haydn's pieces average to a complexity of 1.8860 (standard deviation 0.1693), while Mozart's pieces exhibit a mean complexity of 1.9465 (standard deviation 0.1613). Taken together, we see a fascinating trend of high complexity in the early Baroque period, to very low complexity in the late Baroque, and a very slight increase in complexity during the Classical period. Obviously, much more data are needed to form any conclusions about the long-term development of complexity and there are also many complicating factors, such as geography. It appears that this complexity measure could be useful as a scalar descriptor of a piece of music.

4.2 Signature Patterns

We have applied our signature pattern finders to each of the aforementioned composers. Our early Baroque composers, Rameau and Couperin, present very few signature rhythms amongst their compositions. Any recurring patterns seldom appear in more than two pieces and are remarkable for their irregular onset times suggestive of ornamentation and grace notes. Essentially no signature intervallic patterns were found for either composer. This paints the picture of composers who created variety with novel rhythms and melodies, without specific repetition or development thereof.

The change from the early Baroque period to the late Baroque is noteworthy. While Rameau and Couperin seldom reuse any rhythmic or melodic ideas, the studied works of Handel and Bach contain many rhythmic and melodic patterns used in separate pieces. From 109 keyboard pieces composed by Handel, our software located 504 rhythmic patterns and 633 intervallic patterns repeated between pieces. Some of these signatures were used in nearly forty different pieces, or almost 40%! The patterns themselves are quite illuminating. As might be expected, Bach's favored rhythmic patterns use constant eighth or sixteenth note rhythms. His favorite melodic patterns are mostly scalewise motion, with the most frequent non-scalewise pattern taking the form of a simple arpeggio.

Classical composers Haydn and Mozart also present hundreds of repeated rhythmic and melodic ideas — The two composers speak the same rhythmic language. Melodically, however, we begin to see some differences. Mozart's repeated melodic patterns vary in note length between three and eight notes, while nearly all of Haydn's signature melodic patterns are exactly four notes long. Haydn is more prone to use simple arpeggios than Mozart as well. Finally, we found one curiosity: one of Haydn's favorite melodic patterns comprises a note, then descending exactly one octave and repeatedly playing the lower note. Mozart also enjoys this pattern, however his version always takes on an ascending format: playing one note, then repeatedly playing the note one octave *above* the original note.

The possibilities of signature patterns are very promising. With so much cross-pollination of musical ideas between pieces, if we were to more carefully analyze the vast amounts of data, we expect we would find certain rhythmic or melodic ideas that are completely unique to a certain composer.

4.3 Harmonic Landscape

Finally, we aggregate the harmony distributions of each composer to generate a major and minor harmonic landscape for each. These aggregated histograms represent the probability distribution of each harmony for a composer. The relative usage of each chord can be easily observed in the histograms.

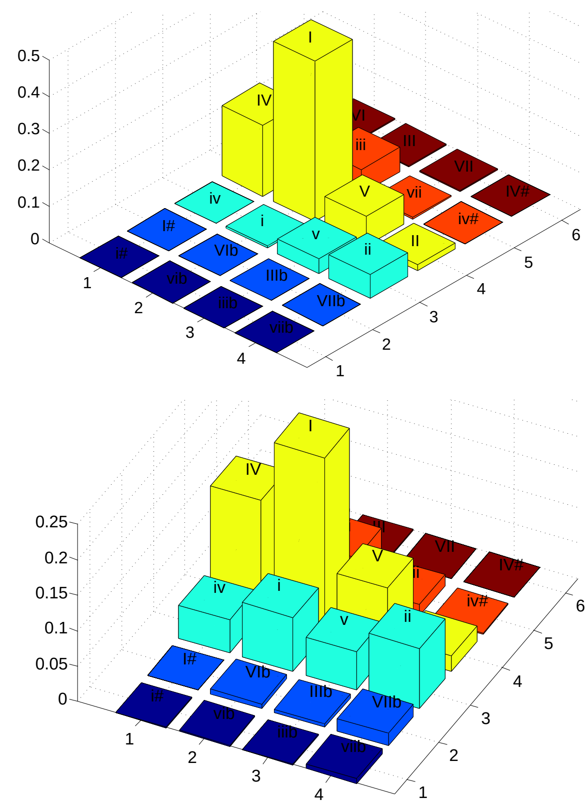

Figure 4: The major harmonic landscapes of Rameau (top) and Couperin. Note the steepness of the histograms. Both are heavily centered on the home key I.

As before, we begin with the early Baroque composers Rameau and Couperin. The most striking feature of the distributions shown in Figure 4, especially Rameau, is the steepness of the histogram.

The home key (I) accounts for nearly half of the harmonic distribution in Rameau. Distant keys are completely unrepresented. Another curiosity is the prominence of the major fourth (IV) and the absence of the major fifth (V). In more modern music, the major fifth is a very important chord which is used to flexibly move between keys. Progressions involving the major fourth have a more ancient sound. In fact, the fourth is a very important chord in psalms and religious chanting.

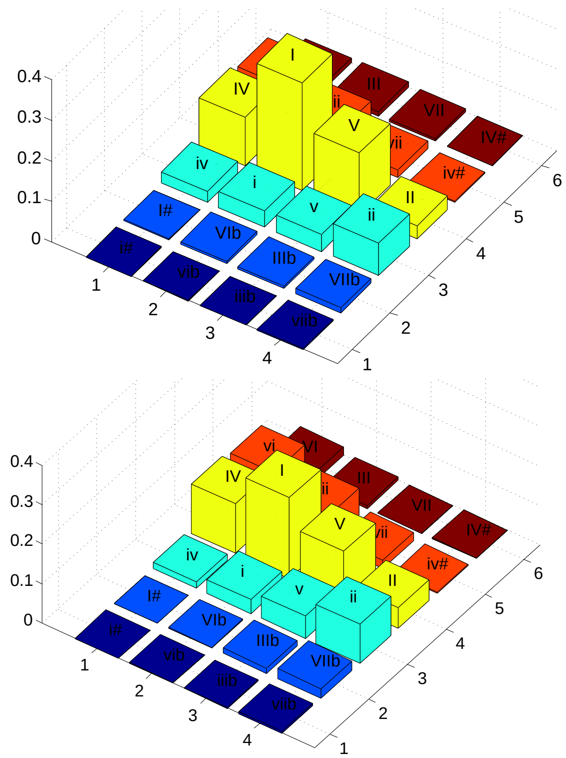

Figure 5: The major harmonic landscapes of Handel (top) and Bach. Note that the major fifth (V) and the major fourth (IV) now rival each other in usage. Observe also that the distributions are less heavily centered on the home key (I).

Investigating the late Baroque major-key works of Handel and Bach, we begin to see some development in the distributions shown in Figure 5. The major fifth (V) and the major fourth (IV) now rival each other in use. There is more exploration of distant keys, in particular the minor second (ii) and the relative minor (vi) are becoming prominent.

In minor pieces, we find some more surprising results. As seen in Figure 6, the minor fourth (iv) is again used significantly more than the fifth (either

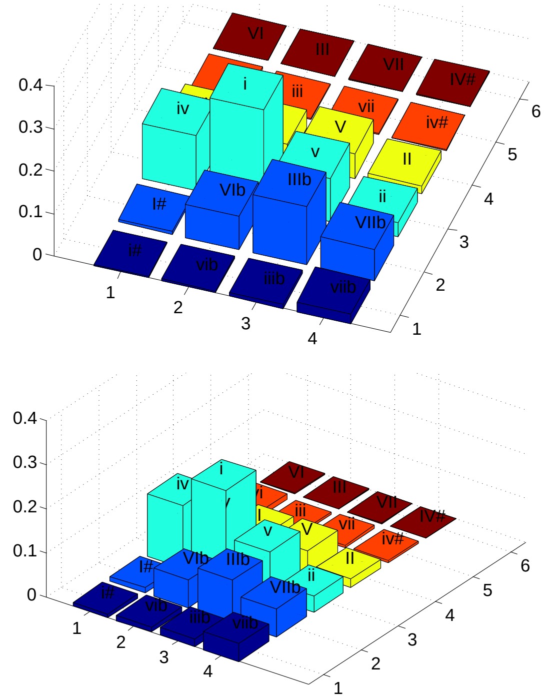

Figure 6: The minor harmonic landscapes of Handel (top) and Bach. Note the exploration of nearby major keys on the part of Handel. Observe also that the minor fourth (iv) is used much more than the fifth (either V or v).

V or v), revealing that the composers were more comfortable with an ancient sound in their more somber pieces. We also see extensive exploration of nearby major keys.

Finally, for Haydn and Mozart the distributions shown in Figure 7 represent a more modern sound, as the major fifth (V) has finally overtaken the major fourth (IV) in usage. There is also exploration of many nearby keys, most prominently the minor second (ii) and the relative minor (vi) again.

The minor harmony distributions of Haydn and Mozart (Figure 8) are remarkable for being very spread out. In particular, Haydn spends almost as much time in the relative major ($III^{\flat}$) as in the home minor key (i). We begin to also see the modern practice of using the major or dominant fifth (V) in minor compositions rather than just major compositions.

5. CONCLUSIONS AND AVENUES FOR FURTHER RESEARCH

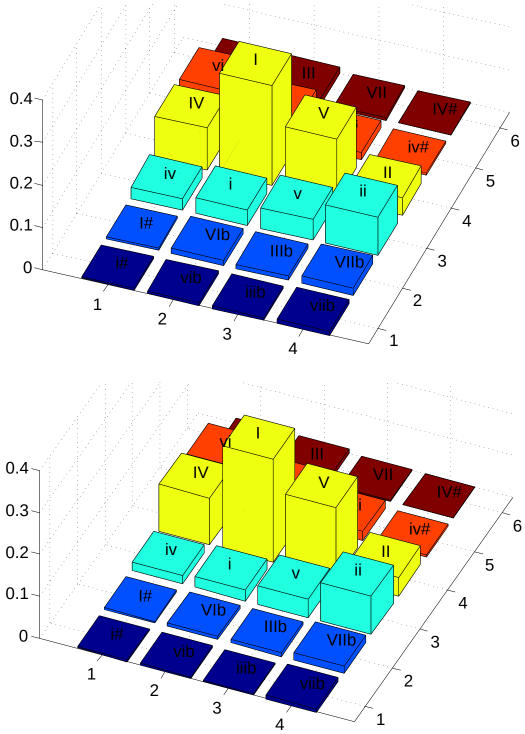

Figure 7: The major harmonic landscapes of Haydn (top) and Mozart. Note the increased prominence of the major fifth (V) and the extensive exploration of nearby keys.

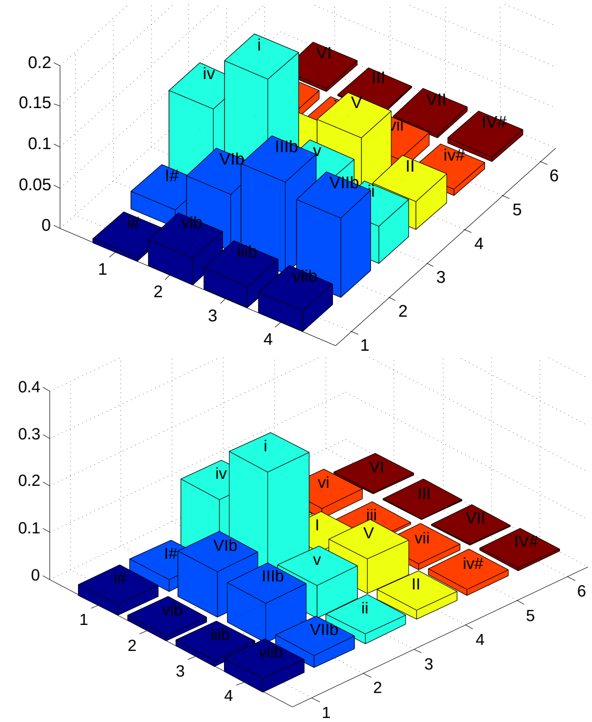

Figure 8: The minor harmonic landscapes of Haydn (top) and Mozart. Note the growing prominence of the major fifth (V) and the use of nearby major keys.

The focus of this research has been to develop flexible tools to simplify further research. The MATLAB programs we developed for this purpose can be found at [10]. We believe we have succeeded in this aim and have only conducted preliminary inquiries with the tools to test their usability. The potential for further study is both wide and fascinating.

The calculation of the complexity of a piece is exceedingly simple and quick, because repeated nested loops are not required as for the signature pattern finders. Running many pieces through the program, we can formulate a better idea of the changes in complexity over time, as we began to see above. Additionally, small adjustments of the program allow us to compare the complexity of each individual voice or only the melodic entropy. We could also consider harmonic entropy using the distribution of the twenty-four major and minor harmonies.

The pattern finder tools can be adjusted to allow researchers to search an entire collection of works for specific rhythmic, melodic, or harmonic patterns of interest. If one does not have a specific target in mind, the programs can adaptively create these targets from the pieces themselves. This allows us to very easily and quickly catalog even the subtlest patterns of a composer within a piece.

The signature pattern finders are more promising still. By supplying the programs with ever more pieces by the same composer, we can say with more and more confidence which patterns were preferred by that composer. As noted above, signature patterns could be invaluable in distinguishing between two similar composers such as Haydn and Mozart.

The algorithm for finding the chord of best fit can be improved. Currently, it can only identify the twenty-four major and minor keys. More modern music frequently uses other chord qualities, notably chords of four notes. Most important among these is the dominant seventh chord (given by the intervals: major third, minor third, minor third). A similar psychological study to that of Krumhansl and Kessler [3] could be used to create reference distributions for other chord qualities than the twenty four major and minor chords. This would allow more precision in our harmony identification (as it is, dominant chords tend to manifest as either the major chord that makes up the bottom half of the chord or the minor chord that makes up the upper half).

A potential goal for the future would be the careful collection of reference data from many different composers. From this, we hope to be able to correctly guess the era or composer of a mystery piece. Given an unknown MIDI file, we can calculate its complexity, find any repeated patterns, and generate its harmonic landscape. Each of these descriptors can then be compared to our collection of reference data to find the best match for era or composer.

Two further questions might explore the social and geographic development of western music. Presently, we have only endeavored to note the changes in complexity, signature patterns, and harmonic construction over time. Another avenue of inquiry would be to study how these variables change geographically in the same time period. Does Classical French music have a different characteristic harmonic distribution than Classical Germanic music?

Historically, some composers, such as Bach, were known to compose pieces on commission. These pieces often took the form of expansions or variations on a previously existing melody and were composed rather quickly. We wonder if we can distinguish between these commissioned pieces and pieces composed over a long period of time with the complete attention of the composer. Might these uncommissioned pieces contain more complex musical ideas or more signature touches? Perhaps they would contain more adventurous harmonic ideas?

We are also interested to apply our programs to modern popular music. The MIDI files are readily available for most popular songs of the past fifty years. Many popular music software today such as Apple's Genius and Pandora Internet Radio have features which suggest artists considered similar to other musical artists. Our musical analysis programs could provide quantitative comparisons between two different artists or two different songs.

Overall, the current work shows great promise and suggests many further applications, refinements, and developments.

6. REFERENCES

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[9] Most midi files used in this work were obtained at `http://www.kunstderfuge.com` and `http://qq.themefinder.org/`

[10] The suite of matlab programs we developed for this project can be downloaded from `http://people.kzoo.edu/barth/music_matlab`