

## Answers for Chapter 5

- 5.1 Eclipses (both solar and lunar) are examples of the projection of an object onto a non-planar surface. All the maps in an atlas are examples of the use of curved projectors. If the projectors were not curved we could not project the entire surface of a spherical object to a rectangle.
- 5.2 Suppose the axis of the plane is its z direction and up is the y direction. In the airplane's coordinate system, the roll, pitch, and yaw correspond to rotations about the z, x, and y axes respectively. We control the orientation relative to the origin by a rotation of the form  $R_z(\text{roll})R_x(\text{pitch})R_y(\text{yaw})$ . We also have to do a translation to move the airplane to its desired location.
- 5.3 Suppose we want the view of the earth rotating about the sun. Before we draw the earth, we must rotate the earth which is a rotation about the y axis. Next we translate the earth away from the origin. Finally we do another rotation about the y axis to position the earth in its desired location along its orbit.
- 5.6 Start with a translation of the COP to the origin. Then it remains to find the intersection of a projector from the origin to a point  $(x, y, z)$  with the plane  $ax + by + cz + d = 0$ . Any point on this projector can be written as  $(\alpha x, \alpha y, \alpha z)$ . Substituting into the equation of the plane, we get  $\alpha = (-d)/(ax+by+cz)$  and the point of intersection is  $\alpha(x, y, z)$ . We can use the homogeneous coordinate form to compute the denominator as the 4<sup>th</sup> coordinate, yielding the matrix:  $P = \frac{x(by + adx) - dx(d - by - cz)}{adx + bdy + cdz}$
- 5.7 The result follows from the transformation being affine. We can also take a direct approach. Consider the line determined by the points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ . Any point along the line can be written parametrically as  $\alpha x_1 + (1 - \alpha)x_2 + \alpha y_1 + (1 - \alpha)y_2$ . Consider the simple projection of this point which is of the form  $f(\alpha)$   $(\alpha x_1 + (1 - \alpha)x_2 + \alpha y_1 + (1 - \alpha)y_2)$ . This form describes a line because the slope is a constant. The function  $f(\alpha)$  implies that we trace out the line at a non-linear rate as  $\alpha$  increases from 0 to 1.

- 5.17 All the points on the projection of the point  $(x, y, z)$  in the direction  $(dx, dy, dz)$  are of the form  $(x + \alpha dx, y + \alpha dy, z + \alpha dz)$ . Thus the shadow of the point  $(x, y, z)$  is found by determining the  $\alpha$  for which the line intersects the plane:

$$ax_s + by_s + cz_s = d$$

Substituting and solving,

$$\alpha = \frac{d - ax - by - cz}{adx + bdy + cdz}$$

We need a projection matrix. Using this value of  $\alpha$ , we get

$$x_s = z + \alpha dx = \frac{x(bdy + cdz) - dx(d - by - cz)}{adx + bdy + cdz}$$

with similar equations for  $y_s$  and  $z_s$ . These results can be computed by multiplying the homogeneous coordinate point  $(x, y, z, 1)$  by the projection matrix

$$M = \begin{pmatrix} bdy + cdz & -bdx & -cdx & -ddx \\ -ady & adx + cdz & -cdy & -ddy \\ -adz & -bdz & adx + bdy & -ddz \\ 0 & 0 & 0 & adx + bdy + cdz \end{pmatrix}$$

5.20 Suppose that the average of the two eye positions is at  $(s, y, z)$  and the viewer is looking at the origin. K we could form the images using the LookAt function twice:

```
gluLookAt(x - dx/2, y, z, 0, 0, 0, 1, 0);
//draw the scene here
//swap buffers and clear
gluLookAt(x + dx/2, y, z, 0,0,0,1,0);
//draw the scene again
//swap buffers and clear
```