

CS 334 Opportunity Day
Fall 2004

NAME: _____

PROBLEMS CHOSEN: _____

EXTRA CREDIT: _____

GRADE: _____

GRADE: _____

TOTAL: _____

Directions: Choose 6 problems at 25 points each to make a total of 150 points. You must include at least one from {1, 2}, at least 2 from {4, 5, 6, 7}, at least one from {8, 9}, at least one from {10, 11, 12}, and any additional problem to make a total of 6 problems. Partial credit will be given as appropriate. If time permits, you may choose extra problems for 10 points each, no partial credit.

1. Tell whether each of the following languages is regular or not regular. If a language is regular, draw a diagram that recognizes the language or define a machine that recognizes it. If a language is not regular, prove that it is not regular.
 - (a) $\{ a^x \mid x = 3n + 1 \text{ for some } n: \mathbf{N} \}$
 - (b) $\{ p \mid p \text{ is a palindrome} \}$ Use the alphabet $\Sigma = \{a, b\}$.

2. Tell whether each of the following languages is regular or not regular. If a language is regular, draw a diagram that recognizes the language or define a machine that recognizes it. If a language is not regular, prove that it is not regular.
 - (a) The language consisting of equal numbers of a's and b's.
 - (b) The language consisting of all strings of even length. Use the alphabet $\Sigma = \{a, b\}$.

3. Prove that the set of integers is a regular language. Hint: One way to do this is to design a regular expression for the language over alphabet $\Sigma = \{0, 1\}$ to describe the set of all binary numerals without leading zeros (except 0 itself) i.e., $\{0, \pm 1, \pm 10, \pm 11, \pm 100, \dots\}$. You could also design a FSA to recognize integers in any base of your choice.

4. Convert the following NFA to a DFA:

5. Write a regular expression describing the same language as the FSA:

6. Construct a finite state automaton that recognizes the regular expression $a^* + ab$.
7. Find a minimal state automaton for the following diagram:

8. True or False: Every subset of a regular language is regular. Prove your answer.
9. True or False: Every language over some given alphabet Σ is a subset of a regular language. Prove your answer.
10. Define $\text{Pref}(L) = \{ w \in \Sigma^* \mid x = wy \text{ for some } x \in L, y \in \Sigma^* \}$. This is sometimes called the set of prefixes of L . Show that regular languages are closed under Pref , i.e., if L is regular, then $\text{Pref}(L)$ is regular. HINT: Induction is a good way to do this.

11. Rewrite in another form, simpler if possible: $b + ab^* + aa^*b + aa^*ab^*$

12. Give short answers to the following:

- (a) How much memory does a FSA have?
- (b) True or False: Every FSA can be implemented as an algorithm. Explain.
- (c) True or False: Every algorithm can be implemented as a FSA. Explain.