CS 334 – Fall 2004: Answers for Assignment 6

1. (q, 01, Z_0) (q, ε, XZ_0) (q, ε, XZ_0) (p, ε, Z_0) (p, 1, Z_0) (p, ε, ε)

2. (1) $\delta(q, 0, Z_0) = \{(q, X)\}$ (2) $\delta(q, 0, X) = \{(q, XX)\}$ (3) $\delta(q, 1, X) = \{(p, \varepsilon)\}$ $\delta(p, 1, X) = \{(p, \varepsilon)\}$

3. General Idea: Accept by final state. The final states in this machine are q_1 and q_3. (1) $\delta(q_0, \varepsilon, Z_0) = \{(q_1, Z_0), (q_2, Z_0), (q_3, Z_0)\}$ initial guess (2) $\delta(q_1, c, Z_0) = \{(q_1, Z_0)\}$ Assume there are no a's or b's and consume all c's. (3) $\delta(q_2, a, Z_0) = \{(q_2, XZ_0)\}$ (4) $\delta(q_2, a, X) = \{(q_2, XX)\}$ Use X to count a's. (5) $\delta(q_2, b, X) = \delta(q_4, b, X) = \{(q_4, \varepsilon)\}$ Upon finding b, go to state q_4 and pop X's against b's (6) $\delta(q_4, \varepsilon, Z_0) = \{(q_1, Z_0)\}$ If we reach the bottom of the stack in q_4, there have been equal numbers of a's and b's, so we go to accepting state q_1 where we read all c's and accept. (7) $\delta(q_3, a, Z_0) = \{(q_3, Z_0)\}$ Read all a's and accept. (8) $\delta(q_3, b, Z_0) = \{(q_5, XZ_0)\}$ Upon reading a b, start counting b's and go to state q_5, a non-accepting state. (9) $\delta(q_5, b, X) = \{(q_5, XX)\}$ Count b's (10) $\delta(q_5, c, X) = \delta(q_6, c, X)$ $\{(q_6, \varepsilon)\}$ Upon reading c, go to q_6 and match c's against b's. (11) $\delta(q_6, \varepsilon, Z_0)$ $\{(q_3, \varepsilon)\}$ Upon getting to the bottom of the stack, we have read an equal number of b's and c's. We accept in q_3, but pop the stack so we cannot accept after reading more a's.

4. (q_0, bab, Z_0) (q_2, ab, BZ_0) (q_3, b, Z_0) (q_1, b, AZ_0) (q_1, ε, Z_0) (q_0, ε, Z_0) (f, ε, ε)

5. $S \to 0S1 \mid 01$

6. $S \to AB \mid CD$ $A \to aA \mid \varepsilon$ $B \to bBc \mid E \mid cD$ $C \to aCb \mid E \mid aA$ $D \to cD \mid \varepsilon$ $E \to bE \mid b$

7. Base: No string of length 1 has ab as a substring. Induction Hypothesis: Any string of length less or equal to n cannot have ba as a substring. Consider a string w of length n + 1. Suppose w = αbaβ where α and β are substrings of w, possibly empty. If α is not empty, then αba is a substring of length less than n + 1 with a substring ab, but this contradicts the induction hypothesis. Similarly for β. If α is empty, then w = baβ. Factor β into μγ, each non-empty. Now baμ, a string with length less than n + 1. By induction hyporthesis, baμ cannot have a substring ba. Contradiction.

8. $S \to S + S \mid SS \mid S^* \mid (S) \mid 0 \mid 1 \mid \phi \mid \varepsilon$