Let us use the four steps of induction to prove that \( \sum_{i=1}^{n} \frac{i(n+1)}{2} \).

**Hypothesis:**
For all \( n \geq 1 \), we hypothesize that \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \).

**Base Case:**
Show that \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \) for \( n = 1 \).

\[
\text{LHS: } \sum_{i=1}^{n} i = \sum_{i=1}^{1} i = 1
\]

\[
\text{RHS: } \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1
\]

Thus the LHS and RHS are the same and the equation holds for \( n = 1 \).

**Assumption:**
Assume that \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \) holds true for values of \( 1 \leq n \leq m \).

**Induction:**
Show that it holds true for \( n = m + 1 \), that is show \( \sum_{i=1}^{m+1} i = \frac{(m+1)(m+2)}{2} \).

\[
\sum_{i=1}^{m+1} i = \sum_{i=1}^{m} i + (m+1)
\]

\[
= \frac{m(m+1)}{2} + (m+1)
\]

\[
= \frac{m(m+1) + 2(m+1)}{2}
\]

\[
= \frac{m(m+1) + 2(m+1)}{2}
\]

\[
= \frac{(m+1)(m+2)}{2} \quad \text{QED}
\]
Consider the following code segment which adds the integers in an array.

ALGORITHM: sum_array
input: array a[1..n] of n integers
output: sum of integers in array placed in variable sum

1 sum = 0
2 for i = 1 to n
3 sum = sum + a[i]

Loop Invariant:
Before the start of the \( p \)th iteration of the loop, the variable \( \text{sum} \) contains the sum of the first 1..(\( p - 1 \)) elements of the array.

Initialization:
The loop invariant is true before the first iteration of the loop (true for \( i = 1 \)). Before the iteration of the first pass of the loop, \( \text{sum} \) is initialized to zero which is the sum of the first 0 elements of the loop. Thus the variable \( \text{sum} \) holds the correct sum before the first pass of the loop.

Maintenance:
If the loop invariant is true before the \( p \)th iteration, show that it is true before the \( p + 1 \)st iteration. Before the \( p \)th iteration of the loop, the loop invariant tells us that \( \text{sum} = a[1] + ... + a[p-1] \). During the \( p + 1 \)st iteration, we execute \( \text{sum} = \text{sum} + a[p] \) so that \( \text{sum} \) now holds the sum of the first \( p - 1 \) elements plus the \( p \)th element. This is the sum of the first \( p \) elements of the array. Thus before the start of the \( p + 1 \)st iteration, the variable \( \text{sum} \) holds the sum of the first \( p \) elements of the array.

Termination:
Show the loop invariant gives us a useful property upon termination. At termination, \( i = n + 1 \). Thus before the start of the \( n + 1 \)st iteration, the variable \( \text{sum} \) holds the sum of the first \( n \) elements of the array. This is the desired outcome of the algorithm and hence we have proven that this algorithm sums the contents of the array.