

Lining Things Up

Theorem 1 For all positive integers n ,

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ 2T(n/2) + n^3 & \text{otherwise} \end{cases} \leq dn^3$$

for some $d \geq \max\{c, 4/3\}$.

Proof We will proceed using induction.

Base case: When $n = 1$, $T(n) = c \leq d(1)^3$ for some $d \geq c$.

Induction hypothesis: Assume that $T(k) \leq dk^3$ for all $1 \leq k < n$.

Induction step:

$$\begin{aligned} T(n) &= 2T(n/2) + n^3 && \text{(by definition)} \\ &\leq 2d(n/2)^3 + n^3 && \text{(by the induction hypothesis)} \\ &= (d/4 + 1)n^3 \\ &\leq dn^3 \end{aligned}$$

for any $d \geq 4/3$. □

The truth table for $p \wedge q$ is

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

k	x_1^k	x_2^k	x_3^k	remarks
0	-0.3	0.6	0.7	
1	0.47102965	0.04883157	-0.53345964	*
2	0.49988691	0.00228830	-0.52246185	s_3
3	0.49999976	0.00005380	-0.52365600	
4	0.5	0.00000307	-0.52359743	$\epsilon < 10^{-5}$
7	0.5	0	-0.52359878	$\epsilon < \xi$