1. Consider the following 2 decision problems:

**Partition (PART)**

*Given:* A finite set $A$ and a positive integer size $s(a)$ for each $a \in A$.

*Question:* Is there a subset $A' \subseteq A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a) ?$$

**Multiprocessor Scheduling (MS)**

*Given:* A finite set $A$ of tasks, a length $l(a)$ for each $a \in A$, a number of processors $m$, and a deadline $D$.

*Question:* Can the tasks in $A$ be scheduled on $m$ processors so that the maximum completion time of any task is at most $D$? In other words, is there a partition $A_1 \cup A_2 \cup \cdots \cup A_m = A$ such that

$$\max_i \left\{ \sum_{a \in A_i} l(a) \right\} \leq D ?$$

Carefully prove that the Multiprocessor Scheduling problem is NP-complete by demonstrating a polynomial time reduction from the Partition problem to it. (Show $\text{PART} \propto_P \text{MS}$.)

Pay careful attention to all the details needed in a complete NP-completeness proof.

2. Write backtracking solutions in C++ for each of the following problems. Each of your programs should print an appropriate answer to standard output.

(a) A Hamiltonian cycle in a graph is a cycle that visits every vertex exactly once. The traveling salesperson problem is to find a Hamiltonian cycle with minimum weight in a weighted graph.

Write a member function for your graph class that solves the traveling salesperson problem.

(b) Hi-Q is a solitaire game with a playing board that has little holes in the shape of a cross and 32 pegs (or marbles) that fit into the holes. Starting with the centermost hole open, players move the pegs by jumping one peg over another, either in a horizontal or vertical direction and removing each peg that is jumped over. Diagonal jumps are not allowed. The object is to remove 31 pegs from the board, finishing with a single peg in the center hole. Write a program that finds a sequence of moves that solves this problem.