1. Let $a$, $b$, and $c$ be arbitrary nodes in subtrees $\alpha$, $\beta$, and $\gamma$, respectively, in the left tree in Figure 13.2 in the book. How do the depths of $a$, $b$, and $c$ change when a left rotation is performed on $x$ in the figure?

2. Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, and 8 into an initially empty red-black tree.

3. Show the red-black trees that result after successively deleting the keys 8, 12, 19, 31, 38, and 41 (in that order) from the final red-black tree in the previous problem.

4. Prove that the longest path from a node $x$ in a red-black tree to a descendant leaf has length at most twice that of the shortest path from $x$ to a descendant leaf.

5. Implement a red-black tree template class. Your implementation should follow the same guidelines as the Binary Search Tree and should include the same methods.

6. Include a method in your class that performs the same sort called for in Problem 8 of Homework 2. Does this implementation with a Red-Black Tree appear to be faster than with a Binary Search Tree (assuming your implementations of the two data structures are along the same lines)? Explain your observations in terms of differences between the two data structures.