Introduction to Discrete-Event Simulation

Reference book: “Simulation, Modeling & Analysis (3/e) by Law and Kelton, 2000”

Outline

System, Model, and Simulation

- System: Discrete and Continuous
- Ways to Study a System
- Why Model
- Model Taxonomy
- Why Simulation

Discrete-Event Simulation

- What is Discrete-Event Simulation (DES)
- Example: A Single Server System
- Advancement of Simulation Time
- Components and Organization of Discrete-Event Simulation Model
- Design of Event List

Example: A Single Server System

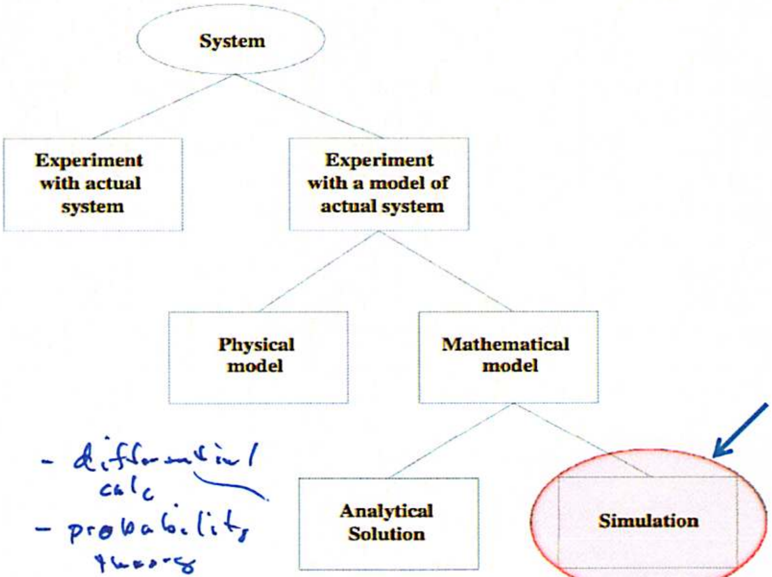
- Sample Design for Event-Scheduling
- Sample Design for Arrival and Departure Event Routine

The Stages of a Simulation Project

System: Discrete and Continuous

- System:
  - a collection of entities that act and interact together toward the accomplishment of some logical end.
- Discrete system:
  - state variables change instantaneously at separated point in time, e.g., a bank, since state variables - number of customers, change only when a customer arrives or when a customer finishes being served and departs
- Continuous system:
  - state variable change continuously with respect to time, e.g., airplane moving through the air, since state variables - position and velocity change continuously with respect to time

Ways To Study a System



System

Experiment with actual system

Experiment with a model of actual system

Physical model

Mathematical model

Analytical Solution

Simulation

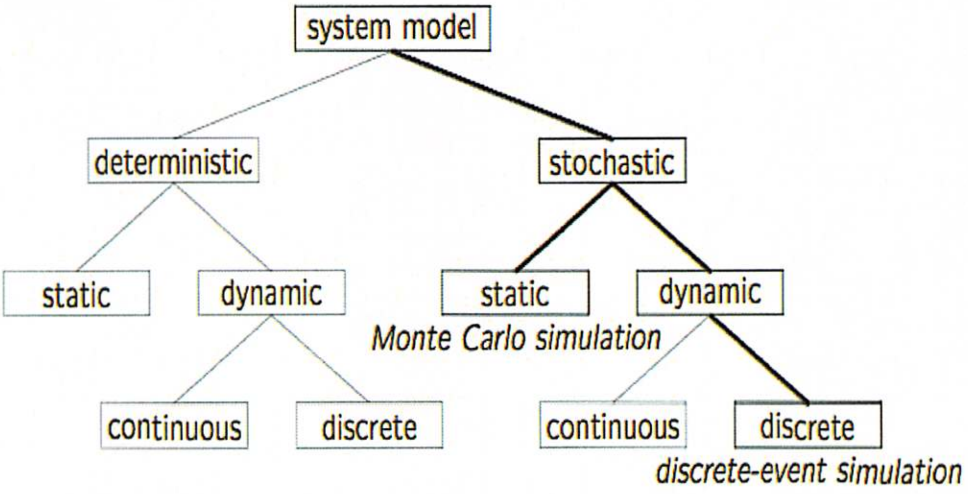
- differential calc
- probability theory
- algebraic methods

Why Model?

- Model:
  - A representation of the system and study it as a surrogate for the actual system
- Why Model?
  - System is not physically exists
  - Building a system is expensive
  - Measuring a system is time-consuming

- Characterizing a Model
  - Deterministic or Stochastic
    - Does the model contain stochastic components?
  - Static or Dynamic
    - Is time a significant variable?
  - Continuous or Discrete
    - Does the system state evolve continuously or only at discrete points in time?
    - Continuous: classical mechanics
    - Discrete: queuing, inventory, machine shop models

Model Taxonomy



Why Simulation?

- Many systems are highly complex, precluding the possibility of analytical solution
- The analytical solutions are extraordinarily complex, requiring vast computing resources
- Thus, such systems should be studied by means of simulation
  - numerically exercising the model for inputs in question to see how they affect the output measures of performance

"Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose either of understanding the behavior of the system or of evaluating various strategies (within the limits imposed by a criterion or set of criteria) for the operation of a system."

-Robert E Shannon 1975

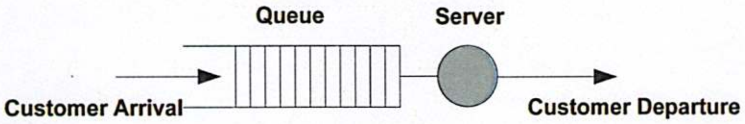
What is Discrete-Event Simulation (DES)

- A discrete-event simulation
  - models a system whose state may change only at discrete point in time.
- System
  - is composed of objects called entities that have certain properties called attributes
- State
  - a collection of attributes or state variables that represent the entities of the system.
- Event
  - an instantaneous occurrence in time that may alter the state of the system

- An event initiates an activity, which is the length of time during which entities engage in some operations
- Entities, attributes, events, activities and the interrelationships between these components are defined in the model of the system

Example: A Single Server System

- Entities: customers; server
- Attributes of a customer: service required
- Attributes of server: server's skill (its service rate)
- Events: arrival of a customer; departure of a customer
- Activities: serving a customer, waiting for a new customer



What is Discrete-Event Simulation (DES)

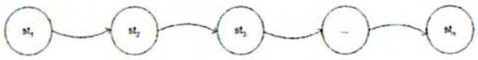
- **Discrete-event simulation is stochastic, dynamic, and discrete**
- **Stochastic = Probabilistic**
  - Inter-arrival times and service times are random variables
  - Have cumulative distribution functions
- **Discrete = Instantaneous events are separated by intervals of time**
  - The state variables change instantaneously at separate points in time
    - The system can change at only a countable number of points in time.
  - These points in time are the ones at which an event occurs.
- **Dynamic = Changes over time**
  - Simulation clock
    - Keep track of the current value of simulated time as the simulation proceeds
  - A mechanism to advance simulated time from one value to another
    - Next-event time advance

Advancement of Simulation Time

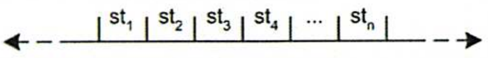
- Fundamental to every simulation study is a mechanism to model the passage of time
- Every model contains a variable called the internal clock, or the simulation clock
- **Time may be modeled in a variety of ways within the simulation**
- How do we advance simulated time?
  - Time as linked events (Next-event time advance)
  - Time divided into equal increments (Fixed-increment time advance)

Advancement of Simulation Time

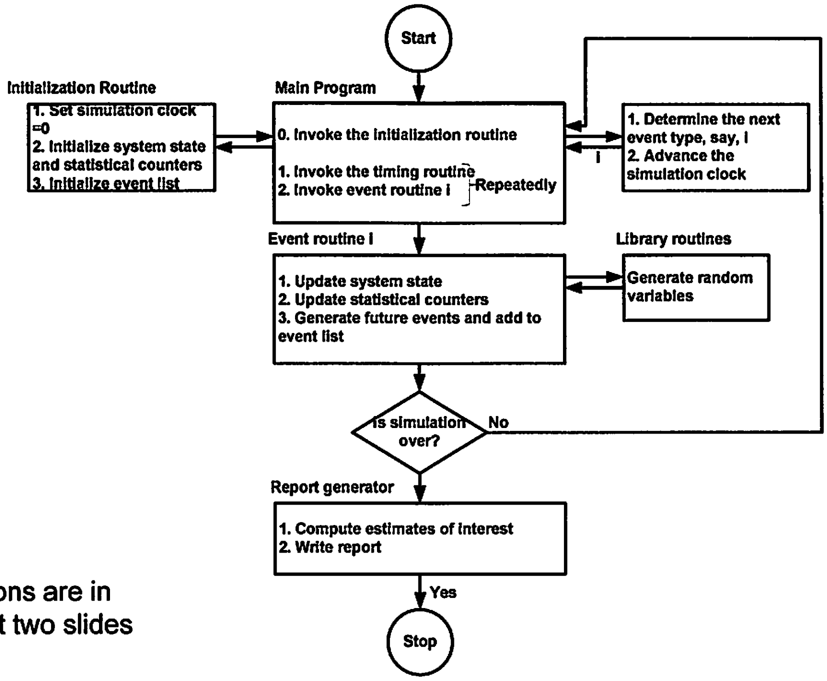
- **Time as linked events (Next-event time advance)**
  - All state changes occur only at event times for a discrete-event simulation model
  - Periods of inactivity are skipped over by jumping the clock from event time to event time
  - This method is called event-driven DES and is asynchronous as opposed to time-stepped approach which is synchronous



- **Time divided into equal increments (Fixed-increment time advance)**



Components and Organization of Discrete-Event Simulation Model



Start

Initialization Routine

1. Set simulation clock =0
2. Initialize system state and statistical counters
3. Initialize event list

Main Program

0. Invoke the initialization routine

1. Invoke the timing routine
2. Invoke event routine i

Repeatedly

1. Determine the next event type, say, i
2. Advance the simulation clock

i

Event routine i

1. Update system state
2. Update statistical counters
3. Generate future events and add to event list

Library routines

Generate random variables

Is simulation over?

No

Report generator

1. Compute estimates of interest
2. Write report

Yes

Stop

Definitions are in the next two slides

Components and Organization of Discrete-Event Simulation Model

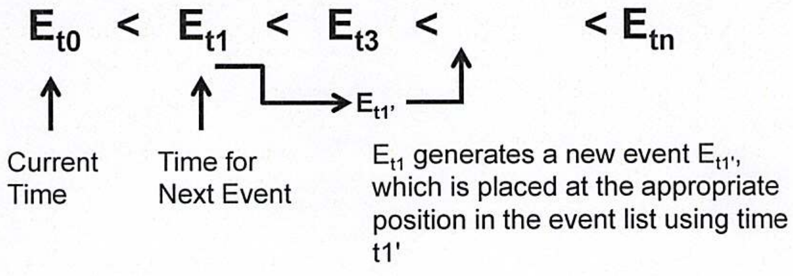
- **System state**
  - The collection of state variables necessary to described the system at a particular time
- **Simulation clock**
  - A variable giving the current value of simulated time
- **Event list**
  - A list containing the next time when each type of event will occur
- **Statistical counters**
  - Variables used for storing statistical information about system information
- **Initialization routine**
  - A subprogram to initialize the simulation model at time 0
- **Timing routine**
  - A subprogram that determines the next event from the event list and then advances the simulation clock to the time when that event is to occur

Components and Organization of Discrete-Event Simulation Model

- **Report generator**
  - A subprogram that computes estimates (from the statistical counters) of the desired measures of performance and produces a report when the simulation ends
- **Event routine**
  - A subprogram that updates the system state when a particular type of event occur
    - There is one event routine for each event type
- **Library routines**
  - A set of subprogram used to generate random observations from probability distributions that were determined as part of the simulation model
- **Main program**
  - A subprogram that invokes the timing routine
    - determine the next event and
  - transfer control to the corresponding event routine
    - update the system state appropriately
  - check for termination
    - invoke the report generator when the simulation is over.

Design of Event List

- Events are chronologically ordered in time.
- Event List
  - sometimes called the pending event set because it lists events that are pending.
  - contains all scheduled events, arranged in chronological time order.
  - In the simulator, this is just a data structure, e.g. list, tree



E_{t0} < E_{t1} < E_{t3} < < E_{tn}

$E_{t1'}$

Current Time

Time for Next Event

E_{t1} generates a new event $E_{t1'}$, which is placed at the appropriate position in the event list using time t1'

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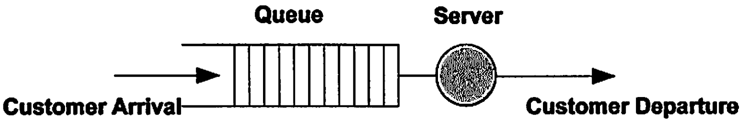
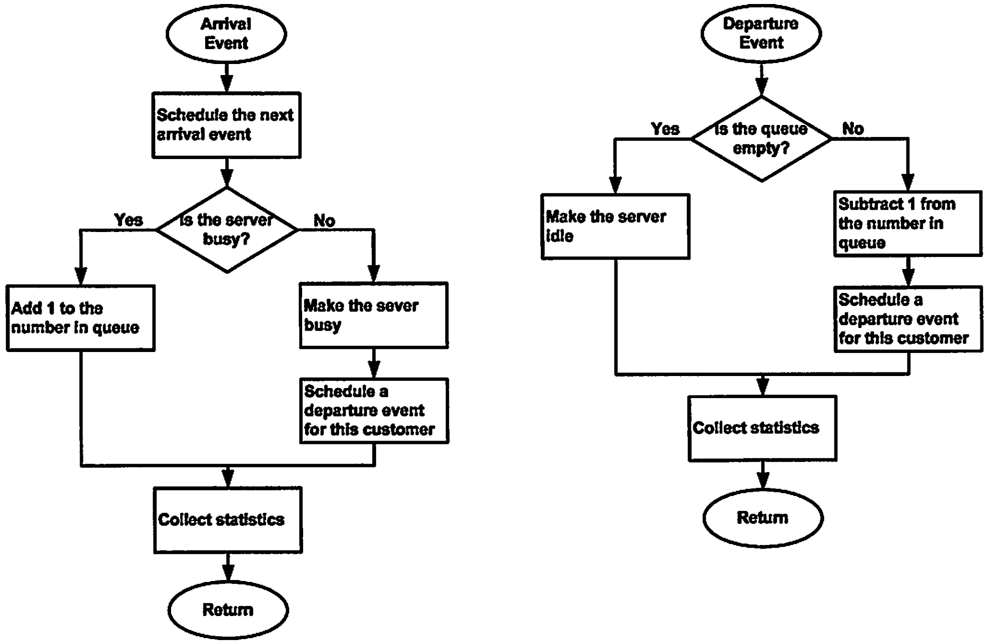


Sample Design for Event-Scheduling

Main (executive routine):
1. set clock = 0
2. set cumulative statistics to 0
3. define initial system state (queue empty, server idle)
4. generate the occurrence time of the first arrival and place in event list
5. select the next event on event list (arrival or departure event)
6. advance simulation clock to time of next event
7. process this event (execute the corresponding event routine)
8. if not end-of-simulation, goto step 5

Sample Design for Arrival and Departure Event Routine



Arrival Event

Schedule the next arrival event

Is the server busy?

Yes

No

Add 1 to the number in queue

Make the sever busy

Schedule a departure event for this customer

Collect statistics

Return

Departure Event

Is the queue empty?

Yes

No

Make the server idle

Subtract 1 from the number in queue

Schedule a departure event for this customer

Collect statistics

Return

The Stages of a Simulation Project

- Planning
  - Problem Formulation: what is it and what do I want to do with it?
  - Resource Estimation: time, people and money.
  - System and Data Analysis
- Modeling
  - Model Building: find relationships.
  - Data Acquisition: find and collect appropriate data.
  - Model Translation: program and debug.
- Verification and Validation
  - Verification: does the PROGRAM execute as intended?
  - Validation: does the PROGRAM represent reality as intended?
- Typically an iterative process

Conclusion

- It is not so hard to write a simulation program
- Efficiency is critical point to a simulation program

Reference

- Simulation, Modeling & Analysis (3/e) by Law and Kelton, 2000
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