A NEWSLETTER FOR THE CALCULUS CONSORTIUM BASED AT HARVARD UNIVERSITY
FOCUS ON CALCULUS

INTRODUCING

The Calculus Consortium Based at Harvard University

Andrew M. Gleason and Deborah Hughes Hallett, Harvard University

We believe that the calculus curriculum needs to be completely re-thought. Under a NSF planning grant we designed a new syllabus, and under our current NSF grant we are writing and testing the materials to support it. In designing the new syllabus, we followed three principles that we believe are important to all calculus renewal projects:

1. Start from scratch. Do not look at the old syllabus and try to decide which topics can be left out. It is much better to take a blank piece of paper and decide which topics are so central that they must be included.

2. Show students what calculus can do, not what it can't do. In a freshman level course, we should be showing students the power of calculus, not the special cases in which it fails. This means, for example, that if we teach Newton's method, we should not emphasize the cases when it doesn't work. We should teach approximations, but not an exhaustive treatment of error estimates. Riemann sums are important, but irregular subdivisions and arbitrary points of evaluation are not. At this stage, complete generality and pathological examples should be kept in the distant background.

3. Be realistic about students' abilities and the amount of time they will spend on calculus. In the past, we taught so much so fast that little understanding was developed. It is far better to teach a few topics well.

"The Rule of Three"

The most difficult and the most necessary aspect of revitalizing calculus is getting our students to think. The old calculus has become a litany of procedures and template problems which too often results only in giving students some rather mindless algebra practice. In addition, students with weak backgrounds are usually driven away by frustration over the manipulations required, even if they are able to understand the basic ideas of calculus.

Changing this state of affairs is not easy because both students and teachers have acquiesced so thoroughly for so long. In order to break the log jam, let's look at what we had hoped students were doing as they learned calculus in the past. Take, for example, the definition of the derivative. First we drew pictures, showing students what the derivative means graphically. Then we talked briefly about the numerical values lying behind the limiting process which takes the secant into the tangent. Lastly we showed students how to compute derivatives analytically. Yet we seldom tested anything other than the analytic aspect, although all the meaning is carried in the first two. We must have believed that students keep with them the graphical and numerical meaning of the derivatives as they work analytically later on. Unfortunately, nothing could be further from the truth. If we ever did ask questions about graphical differentiation or numerical approximations to the derivative, for example, we usually found that an understanding of the graphical or numerical aspects was sadly lacking. Many students who can find derivatives mechanically and solve problems using them often have little idea what a derivative actually means.

Our project is based on our belief that these three aspects of calculus—graphical, numerical, and analytical—should all be emphasized throughout. We call this approach "The Rule of Three" and are working together to design a core curriculum based on this principle. We believe that a course built on this will make a much livelier calculus at any institution—and we represent quite a range. Using the Rule of Three, students will repeatedly be confronted with the graphical and numerical meaning of what they are doing. Besides encouraging understanding, this approach gives students with weak manipulative skills a chance to grasp the concepts behind calculus while strengthening their backgrounds.

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SPRING 1992 ISSUE No. 1

The Core Calculus Course

Our core calculus curriculum is a two quarter or two semester course built around the Rule of Three. The graphical and numerical aspects of each topic are introduced throughout, along with the analytic. For example, using a computer or graphing calculator, the extreme of a function can be found by moving cross-hairs on a graph or by studying the values on a spreadsheet, as well as by the traditional method. Equations can be solved numerically and graphically, as well as algebraically, which also removes the rather artificial condition that they all be either quadratic or factorable. Periodic functions make much more sense to students when presented using a graph or table rather than analytically.

The idea of integration and of solving differential equations comes into our course

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FOCUS ON CALCULUS

NEWSLETTER FOR THE CALCULUS CONSORTIUM
BASED AT HARVARD UNIVERSITY

ISSUE No. 1 SPRING 1992

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Readers are encouraged to share their experiences in changing the way calculus is taught by contributing articles for publication in future issues. Please address your correspondence to Editor, FOCUS ON CALCULUS, and mail it to Susan Elbe at John Wiley & Sons, Inc.

The text materials being written by the Calculus Consortium based at Harvard University (CCH) are being published by John Wiley & Sons, Inc. A preliminary edition will be available for test site use in the fall semester, 1992. For information regarding this text, contact your local Wiley representative, or call the CCH Hotline at (212) 850-6700, ext. 5727.

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FROM THE PUBLISHER

Welcome to the first issue of FOCUS ON CALCULUS. We at Wiley are pleased to have been chosen to publish the text materials written by the Calculus Consortium based at Harvard University (CCH). We are eager to support the consortium members and other calculus educators as they work to transform the way calculus is taught. FOCUS ON CALCULUS is a newsletter for everyone interested in calculus reform, and we encourage you to send in the coupon below so we may keep you on our mailing list.

This premier issue reports on experiences over the past two years with the CCH materials. Consortium schools are represented here, as are test site institutions who have been using the materials this school year. More test sites are being sought for the 1992-93 school year to use the preliminary edition of this text. If you would like more information regarding the CCH materials or participating as a test site, please contact your Wiley representative or call the CCH Hotline at (212) 850-6700, ext. 5727.

Wiley's participation as a sponsor of a two-day conference on the teaching of calculus (see story below) is an important way in which we are supporting growth in calculus education. As the publisher of leading texts in calculus, we look forward to bringing the next generation of top-quality teaching and learning materials to you and your students.

Conference on the Teaching of Calculus

The Calculus Consortium based at Harvard University (CCH) will host a summer conference in conjunction with the NSF and John Wiley & Sons, Inc. on June 12 & 13, 1992. A program of invited speakers, panels, contributed papers, and workshops should provide something of interest for everyone involved in changing the way calculus is taught. The scope is broad; there will be no focus on one particular project, approach, or technology. Two-year college, four-year college, university, and secondary faculty are all welcome.

A preliminary list of speakers assembled by Conference Co-Chairs Tom Tucker (MAA) and Spud Bradley (AMS) includes Lida Barrett, Ronald Douglas, Wade Ellis, James Glimm, Wayne Roberts, and Jerry Uhl. Panels are planned on the following topics: Changing the Climate, Client Disciplines, Nontraditional Classroom Methods, Student Projects, Secondary Schools, and Technology.

Parallel fifteen minute sessions are available for contributed papers. The title of the paper and a 25 word abstract should be submitted by April 15 to Karen or Joe Thrash, Department of Mathematics, SS Box 5045, University of Southern Mississippi, Hattiesburg, MS 39406-5045; E-mail: kthrash@usmcp6 (bitnet). Selected candidates will be notified by May 7.

Attendance will be limited and a $40 registration fee will be charged. Graduate students will be charged a $10 registration fee. To register, or to obtain details on the conference, please call the CCH Hotline at (212) 850-6700, ext. *5727*, or complete and return the coupon below. If you have specific questions, you may call Laura McGayhey at (212) 850-6530.

CONFERENCE REGISTRATION AND SUBSCRIPTION COUPON

❑ **Yes! Please send me detailed information about the Conference on the Teaching of Calculus to be held at Harvard University on June 12 & 13, 1992.**

❑ **Yes! Please register me for the Conference.**
I understand that details of the conference will be sent upon receipt of my registration. Should space be unavailable, my registration fee will be refunded without delay. I am a (check one):

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❑ Graduate Student ($10 registration fee)

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early. We find that slope fields provide a way of introducing the integral which makes it clear graphically that integration reverses the process of differentiation. In addition, using a slope field makes the Fundamental Theorem of Calculus quite transparent.

Of course, if our good intentions are to have much effect, our tests as well as our presentation must reflect the Rule of Three. In addition, it is important to include some non-routine problems regularly. Even if such problems are not hard or long, we must start to reestablish the idea that such problems are not only part of mathematics, but, indeed, the point of mathematics.

Consortium Members

The problems of undergraduate calculus instruction in the United States are of sufficient magnitude that finding a solution will require the efforts of a large number of institutions working in cooperation on a variety of solutions. Our own project involves people from eight institutions:

- Andrew M. Gleason and Deborah Hughes Hallett, *Harvard University,*
- William McCallum, David Lomen, and David Lovelock, *University of Arizona,*
- Andrew Pasquale, *Chelmsford High School,*
- Thomas Tucker, *Colgate University,*
- Jeff Tecosky-Feldman, *Haverford College,*
- Brad Osgood, *Stanford University,*
- Joe Thrash and Karen Thrash, *University of Southern Mississippi,* and
- Sheldon Gordon, *Suffolk County Community College*

During the 1990-91 school year, preliminary versions of the core Calculus Curriculum materials were taught at the consortium institutions. This year, the materials are being class-tested at all the institutions in the consortium and about 20 others. We hope to involve many more institutions in coming years.

A preliminary version of our text will be published by Wiley this fall. If you would like sample materials and information on becoming a test site, please contact your Wiley representative or write to:

Ruth Baruth, Mathematics Editor
John Wiley & Sons, Inc.
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New York, NY 10158-0012

We welcome input and suggestions from the entire mathematics community, and invite its active participation in testing our approach. ▲

TEST SITE REPORT

Middlesex County College

Jacqueline Boyd DeMarzio
Middlesex County College

We used the materials from the Calculus Consortium based at Harvard University (CCH) in two sections of Calculus I last semester at Middlesex County College, a large open-enrollment community college with a very diverse student body in Edison, New Jersey. Two different instructors taught from the materials; one to a class of twenty students, and one to a class of 24 students. This semester, students from those two sections are taking Calculus II and lam teaching that class. The calculus classes meet three times a week—twice for 55 minutes in a regular classroom, and once a week for 110 minutes in the computer laboratory.

Study Groups

Although we have a math tutoring center at our school, I was concerned that the students there, who had not studied calculus under the CCH method, would not be able to help my students. As an alternative, I formed five groups of four students each and met with them to answer any questions they had on homework, or to give them more detailed explanations. The students really enjoyed this—some had gone to the math lab for help unsuccessfully—and although attendance was not required, some students asked if they could attend another group session. I also found that the students met in small groups without me. This semester, I am trying the study groups again, but this time without me.

Student Journals

I also asked students to keep a journal during the course. They were allowed to write anything in the journal that they wished, not just mathematics, and I collected them once a week. I wanted the journals to give me insight into their reactions to the class and any problems they might be having. Some students used the journals to report on alternative solutions they had discovered, or to exclaim about points that they felt they now truly understood for the first time. One student from Kenya wrote:
"As far as the course is concerned, I'm getting the best out of it. The book itself discusses problems in such a way that I feel confident in handling the exercises that follow, even though I may not know all the answers to all the questions. As far as class lecture is concerned, I think it is one of the best methods. Everyone is given the opportunity to demonstrate his or her knowledge of the section under discussion."

Text Reception

One of the complaints I received from students was that there were no answers in the back of the book. I explained that solutions were omitted both to make them think more and because a given problem may have more than one solution. When students were asked to take the integral from a to b of x and explain it geometrically, it illustrated my point. My students were able to come up with three different, correct "solutions" to this exercise. If they had had a solution in the textbook, I don't think this would have been the case.

Students also objected at first to the opening chapter on precalculus. Once we started doing the material, however, their reaction was different. The day we covered compound interest, we worked through the problem and the students came up with the general formula. One student's mouth actually dropped, her eyes grew wide, and she told me that this was the first time that she really understood what she was doing.

When we began worjung with exponential functions and power functions, I got a similar response. One of my favorite problems from Chapter 1 asks students to consider a plane flying from Dulles Airport to LaGuardia Airport that has to circle LaGuardia several times before being allowed to land, and to plot the graph of distance against time for the plane from the moment of take-off in Washington until landing in New York. I asked one student to be Dulles airport, another student to be LaGuardia, and I asked another to go to the board and draw the graph. I acted as the airplane and proceeded to fly from Dulles to LaGuardia, and began circling. As the graph appeared on the board, it was greeted with "oohs" and "ahs" as students grasped the connection between the graph and the physical phenomenon. The method we used contributed to the learning, but I also feel that the problem itself was responsible. Unlike conventional textbook problems, this problem contained no numbers. It asked students to think about the behavior being graphed, and eliminated the possibility of just "doing the numbers."

Technology

I use a computer algebra system and a graphing calculator in my calculus classes. One of the advantages of the graphing calculator

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Middlesex County College

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is the fact that students have it with them at all times. They have it at home when they are doing homework problems, and they have it during classtime when we are not in the laboratory.

In the computer laboratory, I like to interact with my students, so I don't always give them a handout lab. We do problems together and discuss any problems they are having with the solution. I ask for a student volunteer (I usually get quite a few) to come up to the computer at the front and type in the information we're talking about. I then use the blackboard during the discussion.

One such lab asked students to "Sketch a graph of the function $f(x)=x^3-9x^2-48x+52$." When the students had the computer graph this function, they saw two almost vertical lines. At this point, I started asking them to use some of what they had learned to plot this graph. I asked them to find local maxima and minima, global max and mm on a particular interval, and the x-intercepts. We also used a second derivative test, found points of inflection, and analyzed the graph to see where it's concave up, and where it's concave down. Finally, after we had completed the graph, I circled on the board that part of the graph that they had seen at the beginning of the class. They not only enjoyed this exercise, but it stayed with them— a memorable reminder of how to evaluate a function. In addition, it served to reinforce the point that the computer doesn't do the thinking for you, and that to get a meaningful answer from the computer requires understanding. The graph they had seen was accurate, but an incomplete picture of the function.

I always want to bring home to students that the computer or the calculator is a tool to help them, but they have to do the thinking. I recently came up with the idea of assigning problems to groups of three students and asking for a written report from the group. All students get the same grade, and groups are allowed to ask to have students removed if someone is not participating. Despite some early complaints, students now tell me that when one of them doesn't see an answer, another team member does. As they work together to solve the problem and then draft a report that explains their solution method and reasoning, students learn from each other and practice thinking about mathematics. I now find that instead of waiting for me in the hallway before class, students are in the classroom talking in groups when I arrive for class.

TEST SITE REPORT

Success with Lean and Lively Calculus

Gabriella M. Ratay, U.S. Merchant Marine Academy

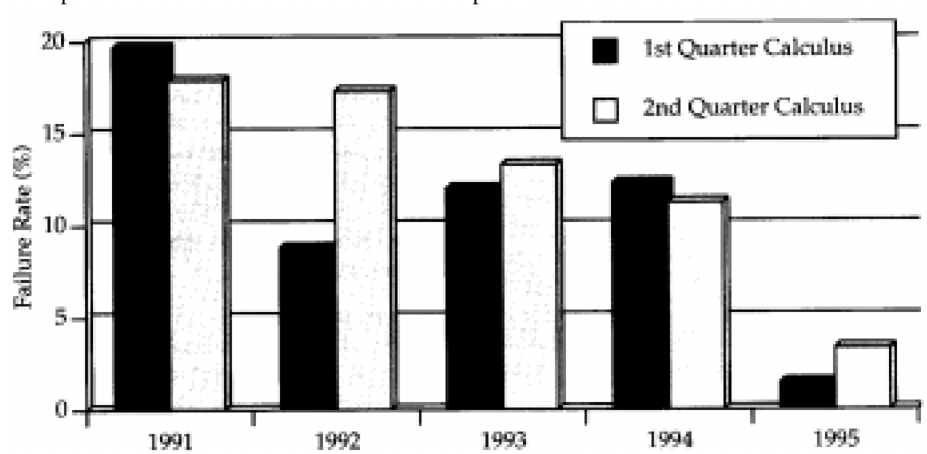
In July 1991, at the start of the new academic year, the mathematics faculty at the U.S. Merchant Marine Academy changed from the traditional method of teaching calculus to the lean and lively approach of the Calculus Consortium based at Harvard University (CCH). The results of the first two quarters are remarkable.

Each year approximately 270 freshmen enter the Academy in two majors Marine Engineering and Marine Transportation. Mathematics is taught to satisfy the needs of major departments and to cooperate with the midshipmen's science education. Students' first math course is calculus, of which they are required to take one full year in four quarters. For the past several years, the high failure rate in calculus had been a serious concern at the institution. When combined with failures in physics and / or chemistry, a number of students were forced to resign.

After using the CCH book and teaching methods, in conjunction with the TI 81 graphing calculator, the student grades for the first two quarters of this year showed a dramatic improvement! The details are shown in the bar graph below: the horizontal axis represents the freshmen by graduating class (this year's frosh will graduate in 1995), and the vertical axis represents the percentage of all grades that were F's.

The graph clearly shows a phenomenal improvement. The percentage of failures with the traditional teaching method ranged from just under 10% to almost 20%; using the CCH approach, the failure rate has been reduced to 1.45% and 3.33% for the first and second quarters, respectively. The quality of the entering freshman class has not changed in the past few years (as measured by SAT scores and an algebra test administered at the start of the school year), neither has there been a significant change in the faculty personnel.

Naturally, it is hoped that by continuing to use the CCH approach this improvement will be maintained in the future. It is also hoped that by emphasizing the numerical and graphical approach as well as the analytical one, that our students will have a better grasp of important concepts and will do better in the courses that depend on calculus. ▲



The Future

I am very pleased with the results I have had with the CCH materials. I have spoken with a number of faculty members at different institutions—universities, four-year schools, two-year schools, and high schools— about the CCH materials and we have formed a New Jersey Calculus Consortium. We have submitted a grant proposal to the NSF that, if approved, will help us implement two pilot sections at each school. If things go well, the method and materials would be adopted department-wide the following year. We are looking forward to the training and support the funding would provide for this exciting project. ▲

TEST SITE REPORT

Brigham Young University

Donald R. Snow, Brigham Young University

Brigham Young University is a private university of 27,000 students with 2000 students taking calculus in any semester. Three faculty members used the materials written by the Calculus Consortium based at Harvard University (CCH) during fall semester 1991 in honors sections of first semester calculus. These 4-credit hour sections met for five 50-minute periods each week. We had little coordination between the sections so this report will only discuss my section.

The self-selected group of eighteen students in my section came from several different majors with only three or four from math. Four had no previous calculus, four had had one previous semester of calculus, and ten had had two previous semesters of calculus. Analysis of the final exam showed that those with one previous semester did better than those with two, and those with no calculus did worse than the other two groups.

Students were assigned to study groups of three or four to meet together regularly, discuss the homework, review for exams, and so on. A couple of assignments were to be done by the study groups. For the most part, the study groups were not effective. Many of the students seemed to get acquainted with other students in the class that were not in their own study group, and they met informally with their friends. Frequently, the class as a whole answered each others questions during class time.

Use of Technology

Students were required to buy an HP-485 or HP-485X graphing calculator (about $270) for the class, which caused a few to drop. We also met in the computer lab one of the five class periods each week to work with IBM PC-compatible computers. I wrote several different handouts for the HP-48, starting with brief instructions for using the machines and the calculus uses for them, and including such topics as computing functions, drawing graphs, using finite differences to approximate derivatives, finding roots, computing Riemann sums and Trapezoidal Rule, and checking the Fundamental Theorem of Calculus numerically. The handouts also suggested problems for the students to try, but I did not collect their work. A few times we spent the entire class answering calculator questions, keying in programs, or discussing how to compute something. The students were expected to bring their calculators to class every day and use them on quizzes and exams. Most of the students learned to use their calculators well and felt at home with them. However, a couple of the students never got the “hang” of modifying programs and could only use the programs I had handed out.

The computer lab was not as successful as the graphing calculators. Again, I wrote and handed out notes on using a spreadsheet in calculus. The handouts were to demonstrate things such as graphs, finite differences, computing integrals by Riemann sums, Trapezoidal Rule, and Romberg corrections based on error analysis of various rules, and verifying the Fundamental Theorem of Calculus. The students expressed interest in using the spreadsheet and noted that they could use it for a lot of other things besides calculus. Students told me they were using it for physics and engineering homework. I didn’t have them hand in any computer work, however, and some students were sporadic in attending the lab at all. This semester, I am having them hand in lab reports.

The students seemed to have more interest in and get more use out of their graphing calculators than from the computers. They spent little extra time in the computer lab. I think the portability and power of the calculator make it a desirable computing device for the students.

Essay Assignments

For the past several years I have assigned two 3-page essays during the semester. For the first semester, one was on the concept of the derivative and one on the concept of the definite integral. Students are told what to assume about the reader and that they will be graded using the “Math Essay Rating Scale” that rates seven different aspects: purpose and rhetorical stance, ideas, mathematical content, arrangement, voice and tone, style, and technical aspects. Some of the students do a really good job on these writing assignments, but others just try to “get by.” I have found that reading these essays brings to light many misconceptions the students have about the material. The students always complain about having to write essays in a math class, however. I think that overall the student essays this semester were a little better than in the past.

The Calculus Consortium Text

I approve of the CCH approach and have been moving in that direction for the past ten or so years. I have been using computers and calculators in calculus since 1980, and the Consortium materials go along with the general way I have been trying to get the students to think about calculus and understand the material.

Toward the end of the semester a girl asked in class, “Are we learning as much

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An Individualized Computer Investigation

Sheldon P. Gordon, Suffolk County Community College

One of the driving forces in calculus reform is the existence of technology that makes many of the standard calculus topics somewhat irrelevant. In the Calculus Consortium materials, we have reduced the emphasis on techniques of integration since virtually any integral that can be evaluated by a student can be done using a computer algebra system, such as Derive, in seconds. Nevertheless, it is important that students not only become familiar with the use of such a system, but also aware of some of the classes of integrals that we no longer cover. To accomplish this, I assign each student in Calculus II a series of individualized antiderivatives that they must evaluate using Derive. The individualization is based on the use of the numerical equivalents of their initials for the values a, b, and c. For instance, if a student’s name is Bill G. Johnson, then a=2, b=7, and c=10.

The particular integrals for this semester are:

$$\int \frac{dx}{a+bx-cx^2} \qquad \int \frac{dx}{\sqrt{a+bx-cx^2}}$$

$$\int x^a \sin(bx)dx \qquad \int \frac{c+a\sin x}{c-b\cos x}dx$$

The students are required to use Derive to evaluate each integral, then to apply the differentiation command to verify that the first result is correct, and if necessary, to apply various other commands to transform the work. They then write up their work and results in a formal report. The report is not only a basis for grading their assignment, but also an opportunity for me to key on any points or ideas that students have missed. ▲

Reform Calculus: Quick Answers to Four Questions

David 0. Lomen, University of Arizona

Will the emphasis on critical thinking and mathematical reasoning in reform calculus diminish basic manipulative calculus skills?

The University of Arizona used the text produced by the Calculus Consortium based at Harvard University (CCH) in all seven sections of its three-unit version of Calculus I during the fall semester 1990. (This class is composed of those students who had the highest scores on our mandatory readiness exam.) Three of the sections were taught by developers of the materials, and the other sections were taught by professors tired of using a standard calculus text.

These students took Calculus 2 spring semester 1991, Vector Calculus (using standard materials) fall 1991, and are currently taking Ordinary Differential Equations. A common 50-minute exam covering basic differentiation, integration, and Taylor series was given the second week of the differential equations class to two-thirds of the sections. No time was spent in class for review, but a handout was given containing basic calculus information and students were encouraged to use the "Are You Ready for ODE's" disk. The backgrounds of the 178 students who took this exam are below.

- 31 students took Calculus 1 & 2 1990-91 using Consortium materials.
- 63 took Calculus 1 & 2 1990-91 using traditional materials.
- 84 took traditional calculus elsewhere or at some other time.

The average scores on this basic skills exam were 83%, 70%, and 70% for each group, respectively.

While the 31 students had higher scores on the readiness exam than did the 63 using traditional materials, we cannot conclude that the consortium materials are responsible for their higher scores. However, we *can* conclude that using consortium materials did not result in a lowering of basic calculus skills.

Will students resent being forced to think, reason, and write more?

Undergraduates taking mathematics classes complete course evaluations. The students are asked to respond to a question with an opinion running from one (agree strongly) to five (disagree strongly). In fall 1991, the question, "I found the course instructive," received 75 ones, 72 twos, 23 threes, 7 fours, and 1 five for the five unit CCH course. The average was 1.80, which was the best of all 13 first and second year courses (covering intermediate algebra through differential equations and linear algebra).

The question, "I would recommend this course to others," received 68 ones, 58 twos, 33 threes, 16 fours, and 3 fives for the five unit CCH course. The average was 2.03, which was again the best average in the 13 first and second year courses.

Will reform calculus result in lower grades?

We have used the CCH materials for all sections of our three unit calculus for two years. The grades earned by students for these two years are shown in the table below (the previous two years are listed for comparison). "Other" includes audits, failures, and drops.

Grades Earned Using Traditional Calculus Text					
	A	B	C	D	Other
Fall 88	25.4%	28.4%	17.8%	8.6%	19.8%
Fall 89	23.9%	35.8%	22.1%	4.0%	14.2%
Grades Earned Using CCH Material					
Fall 90	30.4%	30.4%	22.7%	5.2%	11.3%
Fall 91	28.6%	34.1%	19.2%	12.6%	5.5%

There are a few things to be pointed out in the above table. We first used the CCH materials in fall semester 1990 with our better students, using our better instructors, but with little formal training. Compared to traditional calculus, the grades were generally higher, and the drop/fail rate lower than traditional calculus with no lowering in standards (in fact, some claim a rise in standards). In fall semester 1991 we used a typical mix of instructors and gave them all formal training. The drop/fail rate fell to 5.5%, or about one-third of figure for the traditional calculus course.

Will ethnic minorities be hurt by reform calculus texts?

We are beginning to acquire data comparing the success of minority students taking traditional calculus to those using the CCH material. Here is the evidence so far from the University of Arizona.

Grades Earned Using Traditional Calculus Text					
	A	B	C	D	Other
Fall 90	13%	25%	24%	10%	28%
Grades Earned Using CCH Material					
Fall 91	32%	23%	20%	3%	22%

The grade point average rose from 1.85 in traditional calculus to 2.40 in CCH sections. ▲

Brigham Young University

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using this book as the regular sections of calculus?" I asked if she was understanding what we were doing. She said yes, and that students from other sections were asking her questions about calculus. I said, "And can you answer them?" She said, "Yes. Oh, I guess I am learning as much."

Survey Results

Near the end of the semester I surveyed the students about the course, the text, the calculators, the computer lab, and class time. They liked the book and the HP-48's, but didn't get so much out of the computer lab. Overall, the students thought the course, my quizzes, and exams were too hard, but a few said they enjoyed it very much even though it was hard. Most said they intended to take the second semester of calculus using the CCH materials and the graphing calculators.

It was a good experience for me to teach from the CCH materials, but it took lots of extra preparation time. I am teaching the second semester course from the materials now, and am scheduled to repeat both semesters of calculus starting in the fall. ▲

The Harvard-Hartford Connection

Robert Decker, University of Hartford

In 1988 John Williams and I began to incorporate a laboratory component in our Calculus I and II courses at the University of Hartford, using both graphing calculators and computer software. We had several goals: to show students how calculus can be used to solve real world problems with real data, to expose students to some simple numerical and graphical approaches to problem solving, to get students more involved in the learning process through hands-on work, and to let students explore and discover important calculus concepts themselves. Not much in the way of written materials was available at the time, so we began to write our own lab projects. Our model is similar to that used in the sciences— we go into the lab one day a week, do some experiments (both physical and mathematical), and have the students write up their results in the form of a lab report.

The lab projects evolved into a lab manual, "Bringing Calculus to Life," and we received NSF funding to run workshops for other college and high school calculus teachers in the use of the lab approach. The first workshops were held in 1991; we will hold another set in the summer of 1992. Today there are about 30 schools using our lab manual. At the University of Hartford, our entire department has adopted the lab approach.

We had been using a standard calculus text while developing the lab approach, and I had experienced a certain amount of discontinuity between the text and the lab materials. One student on an evaluation of the course wrote, "I learned calculus better through the labs than through the book." As I was hesitant to write a calculus text myself, I was overjoyed when I came across the preliminary version of the text written by the Calculus Consortium based at Harvard University (CCH). I immediately agreed to use the text in the 1991-92 school year, and to spread the word about it among the schools using our lab manual. Those familiar with the CCH materials will recognize that the goals for the text are quite similar, if not identical, to the ones I listed above for our project. The combination of the two has worked out quite well so far.

Several of the labs involve a simple physical experiment which is performed by two or three volunteers for the rest of the class. Data is collected and then analyzed using the technology. In one lab, students collect time and distance data using stopwatches and yardsticks on a swinging pendulum which is set up against the blackboard. They then have to try to fit the data to the mathematical model, $y=a^{-kt}\cos(wt)$, by experimenting with changes in the parameters a, k, w. In the process, they get a feel for what these parameters represent both graphically and in terms of the pendulum. The model they develop is then used in other labs. Students find times and velocities at various positions, and estimate the time when the pendulum will stop (which can then be checked against the actual pendulum).

In another lab (which has been used successfully at Harvard in conjunction with the new text), students gather data on adult heights using the people in the class. With this data one can then estimate the height of the tallest man and woman in the United States using the normal curve. At the same time, the students must come up with a numerical method for finding proper integrals. Students get to see how the need for new mathematics is related to a real problem situation. In this lab, as well as many of the others, a discussion of sources of error and modeling assumptions becomes an integral part of the process of applying mathematics. Other labs involve using data on the distance of the earth from the sun to estimate the time it takes the earth to get from one point to another on its orbit, modeling the growth of a sunflower plant, and modeling the effect of varying interest rates on investment income.

It is through the process of writing their results in the form of a lab report that the students actually come to grips with what is going on in the project. Though most math students are not used to writing about mathematics, they eventually learn to produce well-written reports.

If you are using one of the standard texts, the lab approach helps to meet several of the goals of the calculus reform movement. If you are using one of the reformed texts (such as the CCH text), the lab component gives the students more in-depth problems along the lines of the ones in your book. Either way, a lab component will help to "bring calculus to life" for your students.

Calculus Reform Funding Opportunities

James Lightbourne, Calculus Program Director, National Science Foundation

As part of an overall plan to strengthen science, engineering, and mathematics education, the National Science Foundation (NSF) has sponsored the Curriculum Development in Mathematics: Calculus Program. Initiated five years ago, the Calculus Program was designed to stimulate the development of projects in response to the need for revision and renewal in the calculus curriculum. The program has provided support for projects that deal with all the topics of one and two-year calculus sequences, including linear algebra and differential equations. While the closing date for the Calculus Program 1992 funding recently passed, there are several other related programs that present funding opportunities to faculty interested in calculus reform.

"Undergraduate Course and Curriculum Development" provides support to plan and develop curricula for introductory level courses with the goal of improving the learning environment in science, engineering, and mathematics for all students. Grants provide for planning, implementation, assessment, and dissemination of projects. (NSF Brochure 91-50; Closes June 15, 1992)

"Undergraduate Faculty Enhancement" offers grants for undergraduate faculty seminars and conferences to provide opportunities for groups of faculty to learn about new techniques and new developments in their fields. Awards are made to conduct seminars, short courses, workshops, or similar activities for groups of faculty members outside the grantee institution. (NSF Brochure 90-112; Closes May 1, 1992)

"Instrumentation and Laboratory Improvement" provides support for projects to develop new or improved instrument-based undergraduate laboratory courses in science, mathematics, or engineering. In addition, the program funds a small number of Leadership Projects in Laboratory Development which provide support for expenses beyond instrumentation in development projects of wide significance. (Closes November 1992 (tent.))

For more information on any of these programs, or to request a brochure, contact the Division of Undergraduate Science, Engineering, and Mathematics Education, Room 639, NSF, Washington, D.C. 20550; Telephone (202) 357-7051; Electronic mail: jhlight@nsf.gov (internet) or jhlight©nsf (bitnet). ▲

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Teaching Reform Calculus to Teachers

Phil Cheifitz and Tom Timchek, Nassau Community College

Nassau Community College is a large two-year college located 30 miles east of New York City, on Long Island. With a mathematics department of over 65 full-time teachers and offering courses from remedial mathematics through linear algebra, the mathematics faculty is a diverse population serving a diverse student body.

In June, 1991, two faculty members became involved with the Calculus Consortium based at Harvard University (CCH). After attending a three-day seminar at Harvard University, we returned to our institution with enthusiasm and hope for a leaner and livelier calculus course, a course that stressed not only analytical analysis but also graphical and numerical analysis of calculus concepts. This new course would use the latest in calculator power, as well as mathematics computer software.

We asked for and received permission to teach from the new materials in four of our twelve Calculus I courses in the fall 1991 semester. Two other faculty members volunteered to teach two of the four classes using the CCH materials. While the students were for the most part not put off by this new approach, we found that a surprising number of faculty were very skeptical of the approach. Many were outright hostile.

As discussions with faculty during the semester progressed, it became evident that there were certain common concerns. We believe that these concerns are not unique to our institution. Among them were:

1. Would the calculator and/or computer replace knowing certain basic "rules"?
2. Could certain topics be omitted from the curriculum without loss of continuity?
3. How would a student who took Calculus I using the CCH materials fare in a traditional. Calculus 2 course, and conversely, how would a "non-CCH" Calculus 1 student perform in a "CCH" Calculus 2 course?
4. How would our large summer calculus enrollment be affected by the new approach?

It also seemed that a very real concern of some faculty (although not explicitly stated) was whether they would be capable of teaching such a "computer-calculator based course, prepared at Harvard."

After we both taught the course we conducted a seminar for our faculty during the winter intercession in order to acquaint our colleagues with exactly how the course differs from the traditional courses. We decided that the best way to introduce our faculty to what we were doing was to actually teach three topics from Calculus 1 during our presentation. We chose to present population growth-exponential functions, the derivative as a linear approximation, and max. mm. problems.

During the presentation we also introduced the use of the calculator and the computer as tools to aid understanding. After the two and a half hour presentation, we believe the 25 attendees felt more confident in their ability to effectively present the material. They saw firsthand that only the "flavor" of the course had changed, and they felt more confident about their ability to learn the new software and integrate it into the course to help reinforce the usual concepts.

We have yet to come to terms with questions 3 and 4 posed above, but we suspect that they will be less of a problem than imagined. ▲

Focus ON CALCULUS

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